Keysight Technologies
Calibration of a Ratio Transformer

White Paper

Abstract

This paper describes a calibration technique for decade ratio transformers which is performed by comparison measurement at 1 kHz. Comparison measurement is made between a device under test and a standard ratio transformer which is calibrated for the first decade by Japan Electric Meters Inspection Corporation. A method using this approach is outlined that uses a commercially available precision capacitance bridge, a four-terminalpair LCR meter, and three-terminal capacitors. Experimental results are given to demonstrate the validity of the calibration technique.

KEYSIGHT TECHNOLOGIES

Unlocking Measurement Insights

Introduction

Decade ratio transformers are used in the calibration of voltmeters, servo components, and devices that require precise division of AC signals in the audio frequency range. Calibration methods for ratio transformers have been described in many papers [1, 2]. Most of the methods are based on comparison measurement between a device under test (DUT) and a standard ratio transformer at each setting using a bridge. These methods require some isolation transformers to reduce stray capacitive loads on the ratio transformers. The stray capacitive loads degrade the incremental linearity of the calibration system due to the output impedance of the ratio transformer. The measurement uncertainty of the DUT is mainly determined from the measurement uncertainty of the standard and the ratio between a signal source output voltage and detector resolution. The calibration method proposed does not require any isolation transformer and is simple. It realizes a semi automatic calibration system using a commercially available precision capacitance bridge.

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Measurement of Output Impedance

Figure 1 shows the equivalent circuit to measure the impedance Z_1. The high-current (Hc) port of a four-terminalpair (4TP) LCR meter applies a signal to a DUT and the high-potential (Hp) port measures the voltage applied to the DUT. The low-potential (Lp) port is designed as a virtual ground point of the low-current (Lc) circuit. The 4TP LCR meter should display the ratio between the sensed voltage *Vm* and current *Im*, that is Z_1.

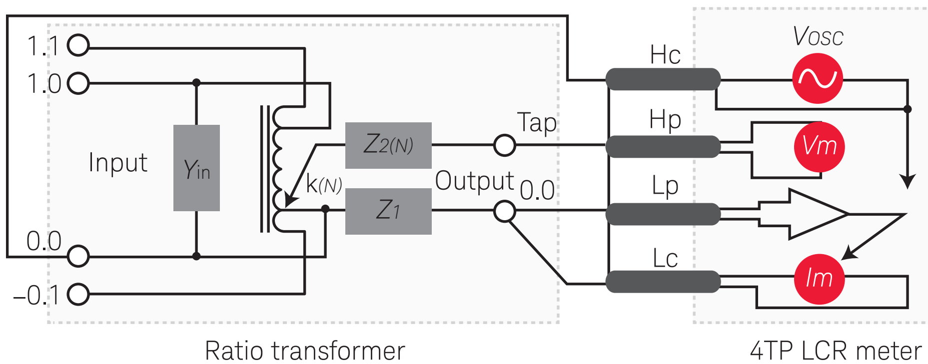

Figure 1. Equivalent circuit for measurement of Z_1.

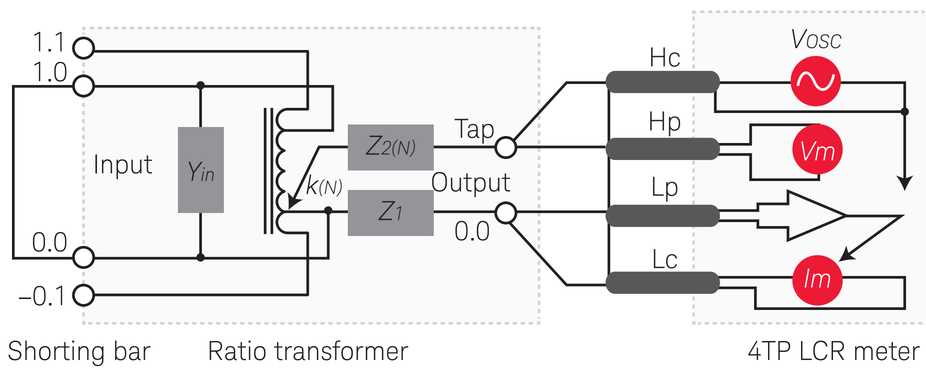

Figure 2. Equivalent circuit for measurement of $Z_{2(N)}$.

For measuring the output impedance $Z_{2(N)}$, the Hc is connected to the Tap and a shorting bar to the input terminals of the ratio transformer in Figure 2. Then the 4TP LCR meter should display the impedance $Z_{m(N)} = Z_1 + Z_{2(N)}$. Then, $Z_{2(N)}$ is given by $Z_{2(N)} = Z_{m(N)} - Z_1$.

The input admittance Y_{in} is measured on the 4TP LCR meter by connecting the Hc and Hp to the input 1.0 and the Lc and Lp to the input 0.0.

Calibration

The technique described in this paper is the comparison of the second decade (from 0.00 to 0.10) of the DUT ratio transformer with the first decade (from 0.0 to 1.0) of the standard ratio transformer at 1 kHz. Other decades of the DUT could be compared with the first decade using a similar method. Figure 3 shows the equivalent circuit of the proposed calibration system.

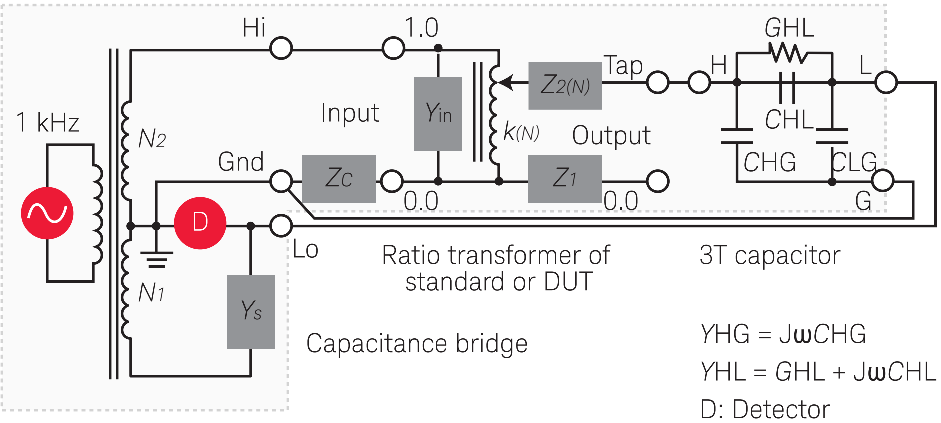

Figure 3. Equivalent circuit for calibration system.

The system consists of the DUT and the standard ratio transformer, the commercially available precision capacitance bridge, and two three-terminal (3T) capacitors. The ratio transformer is connected to a capacitive load that is the 3T capacitor. The output impedance of the ratio transformer $Z_{2(N)}$ is measured and then compensated using the method described in this paper. The standard ratio transformer is calibrated for the first decade by Japan Electric Meters Inspection Corporation (JEMIC). If the capacitance bridge is balanced so that the voltage present at the detector is zero, it should indicate $Y_{m(N)} = N_1 Y_S / N_2$ where N is the nominal ratio of the ratio transformer. A loop equation of Figure 3 can be established by Kirchhoff's laws. By solving it, the ratio of the ratio transformer $k_{(N)}$ is expressed as follows;

$$k_{(N)} \cong \frac{Y_{m(N)}}{Y_{HL}} \left\{1 + Z_{2(N)}(Y_{HL} + Y_{HG})\right\} \left\{1 + Z_C Y_{in} \left(1 - \frac{Y_{HL}}{Y_{m(N)}}\right) + Z_C (1 - N)^2 \, (Y_{HL} + Y_{HG})\right\} \qquad \text{Equation 1}$$

where

$$Z_C \cong \frac{Y_{mC}}{Y_{in} Y_{HL}} \left\{1 + \frac{Y_{mC}}{Y_{in} Y_{HL}} + (Y_{in} + Y_{HL} + Y_{HG})\right\} \qquad \text{Equation 2}$$

and Y_{mC} is the value measured by connecting the H terminal of the 3T capacitor to the input 0.0 terminal. The ratio of the first decade $K_{S(N)}$ calibrated by JEMIC is computed using the following equation which expresses the incremental linearity.

$$K_{S(N)} = \frac{k_{S(N)} - k_{S(0)}}{k_{S(1)} - k_{S(0)}} \qquad \text{Equation 3}$$

STEP 1. Measurement of Standard

Substituting (1) and (2) in (3) gives the ratio obtained from measurement as follows;

$$K_{mS(N)} = \frac{Y_{mS(N)}(1+\Delta_{S1(N)})(1+\Delta_{S2(N)}) - Y_{mS(0)}(1+\Delta_{S1(0)})(1+\Delta_{S2(0)})}{Y_{mS(1)}(1+\Delta_{S1(1)})(1+\Delta_{S2(1)}) - Y_{mS(0)}(1+\Delta_{S1(0)})(1+\Delta_{S2(0)})} \quad \text{Equation 4}$$

$$\Delta_{S1(N)} = Z_{2S(N)}(Y_{HL1}+Y_{HG1}),$$

$$\Delta_{S2(N)} = Z_C Y_{inS}\left(1-\frac{Y_{HL1}}{Y_{mS(N)}}\right) + Z_C(1-N)^2(Y_{HL1}+Y_{HG1}) \quad \text{Equation 5}$$

and Y_{HL1} is the admittance between the low and the high terminal of 3T capacitor used for measurement of the standard ratio transformer.

STEP 2. Measurement of DUT

For the DUT, the ratio of the second decade obtained from measurement is similarly expressed as follows;

$$K_{mD(N)} = 0.1\,\frac{Y_{mD(N)}(1+\Delta_{D1(N)})(1+\Delta_{D2(N)}) - Y_{mD(0)}(1+\Delta_{D1(0)})(1+\Delta_{D2(0)})}{Y_{mD(0.1)}(1+\Delta_{D1(0.1)})(1+\Delta_{D2(0.1)}) - Y_{mD(0)}(1+\Delta_{D1(0)})(1+\Delta_{D2(0)})} \quad \text{Equation 6}$$

$$\Delta_{D1(N)} = Z_{2D(N)}(Y_{HL2}+Y_{HG2}),$$

$$\Delta_{D2(N)} = Z_C Y_{inD}\left(1-\frac{Y_{HL2}}{Y_{mD(N)}}\right) + Z_C(1-N)^2(Y_{HL2}+Y_{HG2}) \quad \text{Equation 7}$$

and Y_{HL2} is the admittance between low and high terminals of 3T capacitor used for measurement of the DUT.

STEP 3. Calibrated Value of DUT

Accordingly, the calibration values of $K_{D(N)}$ are expressed by the following equation.

$$\begin{aligned} K_{D(N)} &= 0.1(K_{S(N)} + K_{mD(N)} - K_{mS(N)}) \\ &= 0.1K_{S(10N)} + 0.1\,\frac{Y_{mD(N)}(1+\Delta_{D1(N)})(1+\Delta_{D2(N)}) - Y_{mD(0)}(1+\Delta_{D1(0)})(1+\Delta_{D2(0)})}{Y_{mD(0.1)}(1+\Delta_{D1(0.1)})(1+\Delta_{D2(0.1)}) - Y_{mD(0)}(1+\Delta_{D1(0)})(1+\Delta_{D2(0)})} \\ &\quad - 0.1\,\frac{Y_{mS(10N)}(1+\Delta_{S1(10N)})(1+\Delta_{S2(10N)}) - Y_{mS(0)}(1+\Delta_{S1(0)})(1+\Delta_{S2(0)})}{Y_{mS(1)}(1+\Delta_{S1(1)})(1+\Delta_{S2(1)}) - Y_{mS(0)}(1+\Delta_{S1(0)})(1+\Delta_{S2(0)})} \end{aligned} \quad \text{Equation 8}$$

Equation (8) is used for the calibration, where N is from 0.00 to 0.10 and Z_{HL2} is selected so that Y_{HL2} approximates $10Y_{HL1}$, (e.g. 10 pF 3T capacitor is used for Y_{HL1} and 100 pF for Y_{HL2}).

Measurement result

Table 1 shows a measurement example of the output and the input impedance of the standard and the DUT ratio transformer. Table 2 shows a measurement example of the admittance and calibrated values of DUT.

Table 1. Measurement example of output and input impedance

N	Standard						N	DUT					
	Z_{1S}		$Z_{2S(N)}$		Y_{inS}			Z_{1D}		$Z_{2D(N)}$		Y_{inD}	
	R1S (Ω)	L1S (µH)	R2S(N) (Ω)	L2S(N) (µH)	GinS (µS)	CinS (nF)		R1D (Ω)	L1D (µH)	R2D(N) (Ω)	L2D(N) (µH)	GinD (µS)	CinD (nF)
0.0			0.03	0.61			0.00			0.03	0.61		
0.1	0.007	0.29	0.50	2.45	0.4	2.86	0.01	0.007	0.29	0.50	2.45	0.4	2.86
1.0			0.03	0.67			0.10			0.03	0.67		

Table 2. Measurement example of admittance and calibrated values

N	Standard				N	DUT			
	KS(N)	YmS(N)		KmS(N)		YmD(N)		KmD(N)	KD(N)
		GmS(N) (pS)	CmS(N) (pF)			GmD(N) (pS)	CmD(N) (pS)		
0.0	0.0000000	0.013	0.000000	0.00000000	0.00	−0.42	0.0000015	0.00000000	0.00000000
0.1	0.0999999	−0.137	1.0000981	0.10000005	0.01	−0.77	0.9999956	0.01000001	0.01000000
1.0	1.0000000	1.19	10.000976	1.00000000	0.10	−2.04	9.9999303	0.10000000	0.10000000

For the calibration of the ratio 0.01, (8) would be rewritten as follows;

$$K_{D(0.01)} = 0.1K_{S(0.1)} + 0.1\frac{Y_{mD(0.01)}(1+\Delta_{D2(0.01)}) - Y_{mD(0)}(1+\Delta_{D2(0)})}{Y_{mD(0.1)}(1+\Delta_{D2(0.1)}) - Y_{mD(0)}(1+\Delta_{D2(0)})} - 0.1\frac{Y_{mS(0.1)}(1+\Delta_{S2(0.1)}) - Y_{mS(0)}(1+\Delta_{S2(0)})}{Y_{mS(1)}(1+\Delta_{S2(1)}) - Y_{mS(0)}(1+\Delta_{S2(0)})} + \delta(0.01)$$

Equation 9

In this case, δ (0.01) that depends on the output impedance of the ratio transformers becomes -1.7×10^{-10} from (8) and (9), and is negligible.

Uncertainty

The measurement uncertainty is estimated by referring the "ISO Guide to the Expression of Uncertainty in Measurement" as follows. For uncertainty estimation for the nominal ratio of 0.01 (second decade), (8) may be approximately rewritten as follows;

$$K_{D(0.01)} \cong 0.1\left[K_{S(0.1)} + \frac{C_{mD(0.01)}}{C_{mD(0.1)}} - \frac{C_{mS(0.1)}}{C_{mS(1)}}\right]$$

$$\cong 0.1\left[K_{S(0.1)} + \frac{C_{mS(0.1)}}{C_{mS(1)}}\left(\frac{C_{mD(0.01)}\,C_{mS(1)}}{C_{mS(0.1)}\,C_{mD(0.1)}} - 1\right)\right]$$

Equation 10

because $Y_{mS(0)}$ approximates $Y_{mD(0)}$ approximates 0, where C_m approximates $Y_m/(2000 \times pi \times j)$, The combined standard uncertainty $u_c(K_{D(0.01)})$ for nominal ratio of 0.01 is the positive square root of the combined variance $u_c^{\,2}(K_{D(0.01)})$, which is given by;

$$u_c^2(K_{D(0.01)}) = \sum\left(\frac{\partial K_{D(0.01)}}{\partial x_i}\right)^2 u^2(x_i)$$

with

$$\frac{\partial K_{D(0.01)}}{\partial x_1} \cong 0.1, \qquad x_1 \equiv K_{S(0.1)}$$

$$\frac{\partial K_{D(0.01)}}{\partial x_2} = \frac{0.1}{C_{mS(1)}}\left(\frac{C_{mD(0.01)}\,C_{mS(1)}}{C_{mS(0.1)}\,C_{mD(0.1)}} - 1\right)$$

$$\cong \frac{0.1}{10pF} \times 2 \times 10^{-6} \cong 20000, \qquad x_2 \equiv C_{mS(0.1)}$$

Equation 11

$$\frac{\partial K_{D(0.01)}}{\partial x_3} = \frac{-0.1C_{mS(0.1)}}{C^2_{mS(1)}}\left(\frac{C_{mD(0.01)}\,C_{mS(1)}}{C_{mS(0.1)}\,C_{mD(0.1)}} - 1\right)$$

$$\cong \frac{-0.01}{10pF} \times 2 \times 10^{-6} \cong -2000, \qquad x_3 \equiv C_{mS(1)}$$

$$\frac{\partial K_{D(0.01)}}{\partial x_4} = 0.1\,\frac{C_{mS(0.1)}\,C_{mS(1)}}{C_{mS(1)}\,C_{mD(0.1)}} \cong 0.01, \qquad x_4 \equiv \frac{C_{mD(0.01)}}{C_{mS(0.1)}}$$

$$\frac{\partial K_{D(0.01)}}{\partial x_5} = 0.1\,\frac{C_{mS(0.1)}\,C_{mD(0.01)}}{C_{mS(1)}\,C_{mS(0.1)}} \cong 0.01, \qquad x_5 \equiv \frac{C_{mS(1)}}{C_{mD(0.1)}}$$

Uncertainty (Continued)

$u(x_1)$: The calibration report gives as the expanded uncertainty of the standard 0.5 ppm and states that it was obtained using a coverage factor of 2. The standard uncertainty is $u(x_1) = 0.5 \times 10^{-6} / 2 = 0.25 \times 10^{-6}$.

$u(x_2)$: Uncertainty of the measured admittance $C_{mS\,(0.1)}$ (approximates 1 pF)
$u(x_{21})$: According to the specification of the capacitance bridge[3] used to measure $Y_{mS\,(0.1)}$, its accuracy is ± 5.5 ppm. The standard uncertainty is
$u(x_{21}) = (5.5 \text{ ppm} \times 1 \text{ pF}) / \sqrt{3} = 3.2 \times 10^{-18}$.

$u(x_3)$: Uncertainty of the measured admittance $C_{mS\,(1)}$ (approximates 10 pF)
$u(x_{31})$: According to the specification of the capacitance bridge, its accuracy is ± 5.1 ppm. The standard uncertainty is $u(x_{31}) = (5.1 \text{ ppm} \times 10 \text{ pF}) / \sqrt{3} = 2.9 \times 10^{-17}$.

$u(x_4)$: Uncertainty of the measured ratio $C_{mD\,(0.01)} / C_{mS\,(0.1)}$ (approximates 1)
$u(x_{41})$: According to the specification of the capacitance bridge, its non-linearity is ± 0.65 ppm. The standard uncertainty is $u(x_{41}) = (0.65 \times 10^{-6}) / \sqrt{3} = 0.38 \times 10^{-6}$.

$u(x_5)$: Uncertainty of the measured ratio $C_{mS\,(1)} / C_{mD\,(0.01)}$ (approximates 1)
$u(x_{51})$: According to the specification of the capacitance bridge, its non-linearity is ± 0.20 ppm. The standard uncertainty is $u(x_{51}) = (0.20 \times 10^{-6}) / \sqrt{3} = 0.12 \times 10^{-6}$.

$u(x_{22})$, $u(x_{32})$, $u(x_{42})$, $u(x_{52})$: Uncertainty due to irregular change during measurement.

Since the data have been obtained as six sets of observation of the four input quantities, it is possible to compute a value for $K_{D\,(0.01)}$ from each set of input data, and then take the arithmetic mean of the six individual values. The experimental standard deviation $s(K_{D\,(0.01)})$ is then calculated from the six individual values. $s(K_{D\,(0.01)}) = 4 \times 10^{-9}$.

The combined standard uncertainty is calculated from equation (11). The individual terms are collected and substituted into this equation to obtain;

$u^2 c\,(K_{D\,(0.01)}) = (0.1)^2 (0.25 \times 10^{-6})^2 + (20000)^2 (3.2 \times 10^{-18})^2 + (-2000)^2 (2.9 \times 10^{-17})^2 + (0.01)^2$
$(0.38 \times 10^{-6})^2 + (0.01)^2 (0.12 \times 10^{-6})^2 + (4 \times 10^{-9})^2$
$= 6.6 \times 10^{-16}$
or $u_c\,(K_{D\,(0.01)}) = 2.6 \times 10^{-8}$.

Conclusion

A calibration technique for decade ratio transformers which is performed by comparison measurement at 1 kHz has been described. It realizes a semi-automatic calibration system using a 4TP LCR meter and a commercially available precision capacitance bridge. The first, second, third, or fourth decade of DUT is calibrated using them. The 4TP LCR meter is used for measuring the input and the output impedance of the ratio transformers. The bridge is used for measuring the capacitance which is divided by the ratio transformers. The combined standard uncertainty for 0.01 is estimated to be 0.026 ppm.

Acknowledgement

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References

1. Svetlana Avramov, Nile M. Oldham, Dean G. Jarrett and Bryan C. Waltrip, “Automatic Inductive Voltage Divider Bridge for Operation from 10 Hz to 100 kHz,” IEEE Trans. Instrum. Meas., vol 42, pp. 131-135, Apr. 1993.
2. “Model 73 Precision Ratio Transformer Instruction Manual”, Electro Scientific Industries, Inc.
3. “2500A 1 kHz Ultra-Precision Capacitance Bridge Operation and Maintenance Manual”, Andeen Hagerling, Inc.