Keysight Technologies
Analyzing Frequency Stability in the Frequency and Time Domains

Application Note

KEYSIGHT
TECHNOLOGIES

Introduction

In many cutting-edge radar and communication systems, frequency stability is the key characteristic that limits system performance. In a radar system, frequency stability affects the processing of Doppler information, and this becomes increasingly important in systems intended to detect slow-moving objects. In a digitally modulated communication system, frequency instability will degrade error vector magnitude (EVM), which is a crucial performance metric.

Random perturbations that show up as instabilities in the frequency or phase of a carrier signal can be caused by a variety of effects. To measure these effects, choosing the most effective and efficient method can be based on the carrier frequency and how close-in its stability must be characterized (e.g., carrier-offset frequency). Short-term frequency stability is most commonly measured in the frequency domain as phase noise, which is defined as single sideband power within a 1-Hz bandwidth at a specific frequency offset from the carrier signal. Stability can also be characterized in the time domain using statistical measures of fluctuations in phase or frequency as a function of time. The most common time-domain methods are Allan variance and the modified Allan variance.

Each approach has advantages and disadvantages, but both perspectives—time- and frequency-domain—may be needed to fully characterize a signal. This application note describes both methods with an emphasis on practical, cost-effective solutions. For those who are new to the time-domain perspective, two proven frequency-to-time conversion methods are also presented.

Working in the frequency domain

Any discussion of phase noise is mostly concerned with the frequency stability of a signal. Long-term stability, perhaps of an oscillator, may be characterized in terms of hours, days or even longer. Short-term stability refers to frequency changes that occur over a period of a few seconds or less. In addition, there are always small, unwanted fluctuations in the amplitude and phase of real-world signals. All of these short-cycle variations have a much greater effect on systems that rely on extreme processing to extract more information from a signal.

Short-term stability can be described in many ways but the most common is single-sideband (SSB) phase noise. SSB phase noise is defined as the ratio of two power quantities: the power density at a specific frequency offset from the carrier and the total power of the carrier signal. This is most commonly measured in a 1-Hz bandwidth at a frequency offset “f” away from the carrier and the units are dBc/Hz or “decibels below carrier frequency power over a 1-Hz bandwidth.”

Figure 1 provides a comparison of two signals, one ideal and one real-world. As expected, the ideal signal appears as a single spectral line in the frequency domain. In contrast, the real-world signal has content spread across several spectral lines above and below the nominal carrier frequency. This content appears as modulation sidebands due to random fluctuations in amplitude and phase.

Ideal Signal

$$V(t) = A_o Sin(\omega_o(t))$$

Where:
A_o = nominal amplitude
ω_o = nominal frequency

Real-World Signal

$$V(t) = (A_o + E(t))Sin(\omega_o(t) + \phi(t))$$

Where:
E(t) = random amplitude changes
Φ(t) = random phase changes

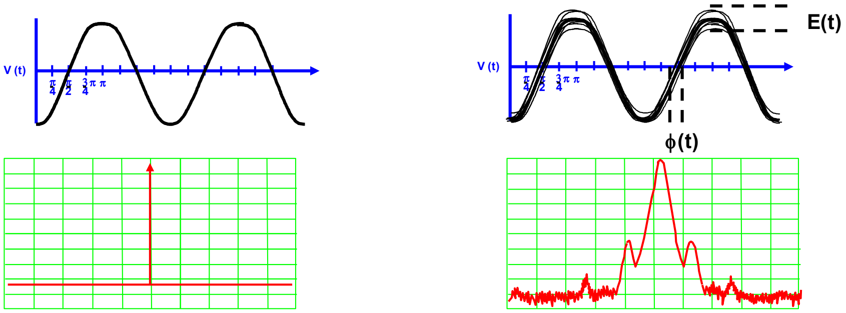


Figure 1. The time-domain equation for the real-world signal (right) includes additional terms that account for the fluctuations in amplitude and phase.

Phase noise measurement techniques have evolved along with advances in spectrum analyzer technology. The rest of this section describes three methods that range from basic to intermediate in complexity: direct-spectrum measurements, phase-detector techniques and two-channel cross-correlation.

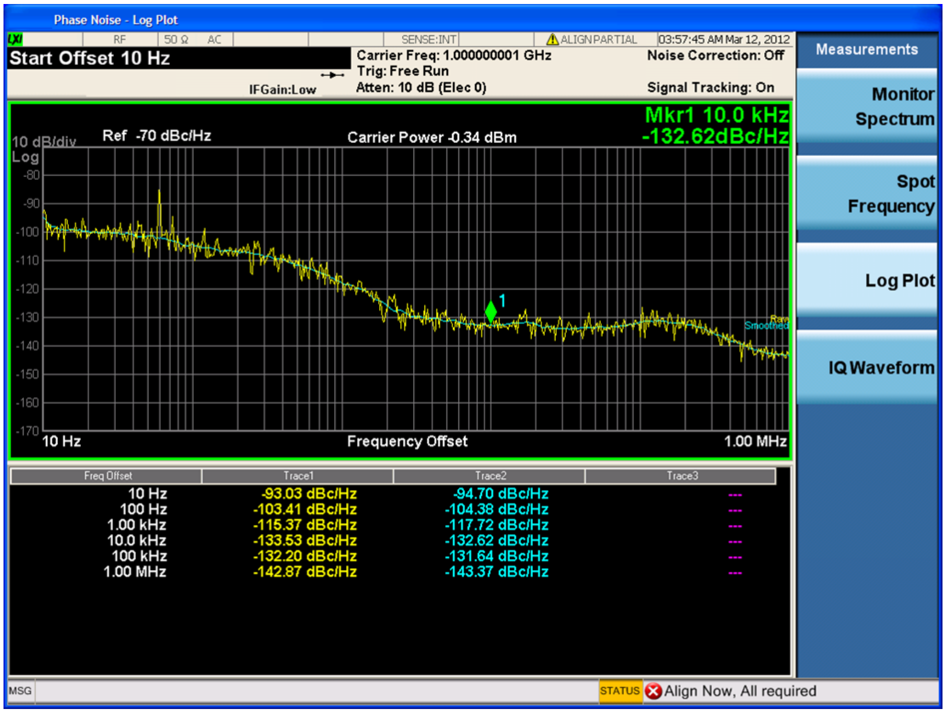
Measuring: Direct-spectrum method

The direct-spectrum approach is the oldest and perhaps simplest way to measure phase noise. The signal-under-test (SUT) or device-under-test (DUT) is simply connected to the input of a signal analyzer such as the Keysight PXA and the analyzer is then tuned to the carrier frequency. Next, two measurements are made: the power of the carrier and the power spectral density (PSD) of the oscillator noise at a specified offset frequency and referenced to the carrier power.

In the absence of specialized features, a variety of corrections must be applied to ensure an accurate result. For example, it may be necessary to correct for the noise bandwidth of the analyzer's resolution bandwidth (RBW) filters. In addition, it may be necessary to also correct for the behavior of the analyzer's peak detector, which may under-report the actual noise power.

It was once necessary to perform these corrections manually, and Keysight Application Note 150, Spectrum Analyzer Basics, is a useful resource. Today, these extra steps are no longer needed when using a signal analyzer equipped with either an interval-band/interval-density marker function (for the PSD measurement) or a built-in capability such as the Keysight N9068A phase noise measurement application for the PXA, MXA and EXA X-Series signal analyzers.

The X-Series phase noise application provides a simple one-button measurement menu that lets the user choose among four modes: spectrum monitoring, spot measurements of phase versus time at a single frequency, a log-plot view, and a display of the I/Q waveform. The log-plot view includes a table of phase noise values at cardinal offset frequencies (Figure 2).



Start Offset 10 Hz

Carrier Freq: 1.000000001 GHz
Trig: Free Run
Atten: 10 dB (Elec 0)

Noise Correction: Off
Signal Tracking: On

Ref -70 dBc/Hz

Carrier Power -0.34 dBm

Mkr1 10.0 kHz
-132.62dBc/Hz

Freq Offset	Trace1	Trace2	Trace3
10 Hz	-93.03 dBc/Hz	-94.70 dBc/Hz	---
100 Hz	-103.41 dBc/Hz	-104.38 dBc/Hz	---
1.00 kHz	-115.37 dBc/Hz	-117.72 dBc/Hz	---
10.0 kHz	-133.53 dBc/Hz	-132.62 dBc/Hz	---
100 kHz	-132.20 dBc/Hz	-131.64 dBc/Hz	---
1.00 MHz	-142.87 dBc/Hz	-143.37 dBc/Hz	---

Align Now, All required

Figure 2. The N9068A phase noise measurement application ensures excellent measurement accuracy by automatically optimizing the measurement in each offset range.

Time and experience have revealed the potential limitations of the direct-spectrum method. Most are related to shortcomings in the quality or performance of some spectrum analyzers: residual FM of the analyzer's local oscillator (LO), the noise sidebands or phase noise of the analyzer LO, and the analyzer noise floor can all affect the results. In addition, most spectrum analyzers measure only the scalar magnitude of the SUT noise sidebands. As a result, the analyzer is unable to differentiate between amplitude noise and phase noise. Finally, the process is complicated by the need to make a noise measurement at every frequency offset of interest—and this is may be very time-consuming when performed manually.

To a large extent, the use of an X-Series signal analyzer and the phase noise measurement application minimizes the effects of the errors described above. For example, when used in the PXA signal analyzer, the AM component of the noise can be removed from the measurement (for offsets of 1 MHz or less). In this case, the analyzer uses its digital IF and I/Q signal processing to measure only the phase-modulated component of the noise, thereby improving measurement accuracy.

To further enhance and simplify the analysis process, the N9068A application also provides special-purpose marker functions that perform a variety of essential measurements. Examples include integrated (RMS) noise, residual FM, averaged noise density, peak search and next spur. Delta-marker functions include absolute, octave slope and decade slope.

In general, the direct-spectrum method works very well with offsets down to 200 Hz. By engaging the default "narrow locking bandwidth" capability of the X-Series analyzers, this technique also works well with offsets in the range of 30 to 200 Hz. The PXA signal analyzer also offers the ability to use an external 10 MHz reference, which may have superior close-in phase noise compared to the high quality reference oscillator built into the PXA. This function can provide almost 10 dB of improvement in measurement accuracy at offsets below 30 Hz.

For enhanced long-term stability, the PXA, MXA and EXA can be configured with the J7203A atomic frequency reference (AFR). The AFR is a portable Cesium-based atomic clock that conveniently attaches to the rear panel external-reference input of the instrument. It also has a plug-and-play USB connection that provides power and simplifies installation and integration.

Reaching millimeter frequencies

A variety of advanced radar systems operate at millimeter-wave frequencies. In this range, system performance is even more dependent on the phase noise of the signal source than when working at microwave frequencies.

To analyze millimeter-wave signals with an off-the-shelf microwave signal analyzer, three hardware solutions are currently available: external harmonic mixers, external "smart" mixers and downconverters. When making phase noise measurements, both -mixer techniques have crucial limitations. The best alternative is downconversion. Depending on the design of the signal analyzer and the downconverter, the configuration requires two or three pieces of equipment: the signal analyzer and downconverter, and perhaps a signal generator to provide a higher-frequency LO signal.

With most downconverters, the LO signal—whether provided by the signal analyzer or a signal generator—is multiplied and amplified before entering the mixer. Depending on the downconverter design, the mixer may perform either fundamental mixing or operate on a low-order harmonic. The latter approach results in lower conversion loss, thereby improving DANL.

For a more on this topic, please see the application note Measurement Considerations for Generating and Analyzing Millimeter-wave Frequencies, publication 5991-3968EN.

Measuring: Phase-detector method

A phase detector can be used to separate phase noise from amplitude noise. As illustrated in Figure 3, the phase detector converts the phase difference of two input signals into a voltage at the output of the detector. When the phase difference is set to quadrature, the voltage will be zero. Any phase variation from quadrature will result in a corresponding voltage change at the output.

This concept is the basis of several commonly used phase noise measurement techniques. Three are of particular interest here: the reference-source/phase-locked loop (PLL) method, the frequency-discriminator method and the heterodyne digital discriminator method. The advantages and disadvantages of each are summarized in Table 1.

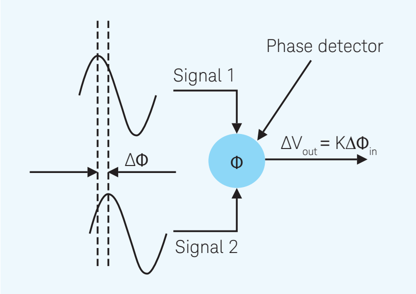


Figure 3. This basic phase-detector concept has been implemented in many ways.

Technique	**Advantages**	**Disadavantages**
Reference-source/PLL	– Best overall sensitivity – Widest measurement coverage – Insensitive to AM noise – Can track drifting sources	– Reference source must have low phase noise – Reference source must be electronically tunable – Must be tunable over wide range if SUT has high drift rate
Frequency discriminator	– Works well with free-running sources such as inductor/capacitor (LC) oscillators and cavity oscillators – Longer delay line can provide improved sensitivity	– Decreased measurement sensitivity, especially at close-in offset frequencies – Longer delay line can reduce signal-to-noise ratio and limit maximum measurable offset frequency – Insertion loss of delay line may be too great compared to SUT level
Heterodyne digital discriminator	– Well-suited to oscillators and unstable signal sources with relative large phase noise – Provides wider measurement range than PLL method – Digital technology eliminates need to connect delay lines used in frequency discriminator method – Enables easy, accurate measurements of AM noise with same setup and RF connections (with delay time set to zero)	– Total dynamic range is limited by the low-noise amplifier and analog-to-digital converters used inside the instrument

For a more detailed discussion of all three methods, please see the Keysight application brief *Exploring Phase Noise Measurement Methods and Techniques,* publication 5991-2069EN.

Measuring: Two-channel cross correlation method

This technique overcomes the disadvantages of the heterodyne digital discriminator method, described in the previous section. Within a measuring instrument such as the Keysight E5052B signal source analyzer (SSA), this approach uses two duplicate reference-source/PLL channels and calculates the cross-correlation between the two resulting outputs (Figure 4).

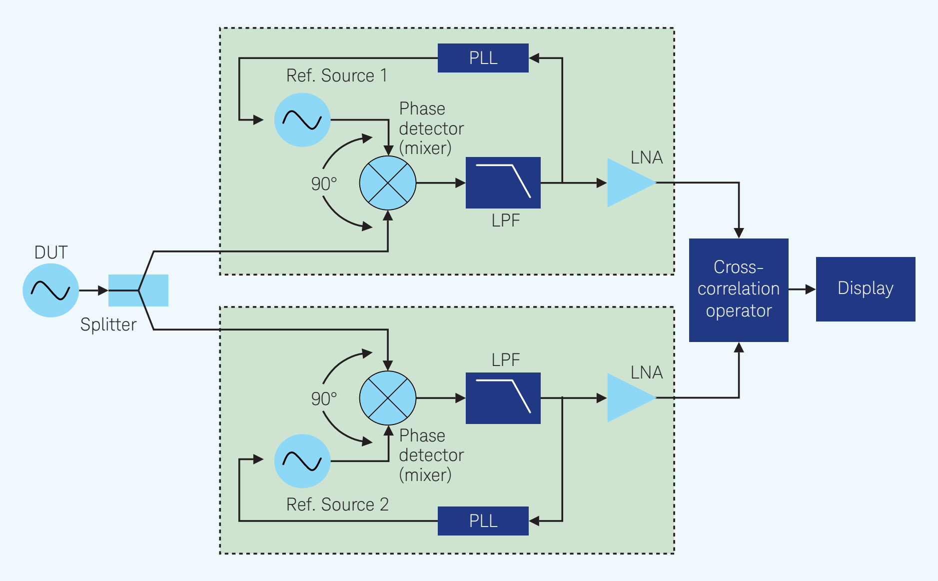


Figure 4. As implemented in the E5052B, the two-channel cross-correlation technique utilizes two phase detectors.

Because any SUT noise present in both channels is coherent, it is not affected by the cross-correlation computation. In contrast, any internal noise generated by either channel is non-coherent and therefore diminished in the cross-correlation operation by the square root of the number of correlations.

The number of correlation operations is a key factor in total measurement time. In the E5052B, the number of correlation operations is a user-selected value. Increasing the number of correlations reduces the noise contribution from both channels (Table 2) but extends the time required to complete the measurement.

Table 2. Increasing the number of cross-correlation operations will reduce the level of non-coherent noise.

Number of corrections	10	100	1,000	10,000
Noise reduction	-5 dB	-10 dB	-15 dB	-20 dB

Because the two-channel technique reduces measurement noise, it provides superior measurement sensitivity and, because it relies on digital signal processing (DSP) capabilities, it enhances sensitivity without requiring exceptional performance in the measurement hardware. This method also provides greater dynamic range than the digital-discriminator method described earlier.

With these benefits, the two-channel cross-correlation method is an especially good choice when characterizing free-running oscillators. As a general observation, it provides excellent phase noise performance when measuring many types of sources and oscillators.

Working in the time domain

When characterizing short-term frequency stability, most engineers are accustomed to the frequency domain, using phase noise plots of dBc/Hz versus offset frequency. However, when working at extremely close-in offsets of 10 Hz, 1 Hz or 0.1 Hz, time-domain measurements can provide additional information and insight.

The first step is to capture a set of gap-free data that is large enough to include—and reveal—gradual changes caused by low-frequency noise. The stability of a signal source is determined by calculating the statistics of its phase or frequency fluctuations versus time, and this is commonly done using variance, which is a second-moment analysis of phase or frequency. The Allan variance is the most widely used calculation; however, it provides relatively low statistical confidence. The most commonly used alternative is the modified Allan variance, which can distinguish white noise from flicker noise.

Either technique can be implemented with a universal frequency counter such as the Keysight 53230A. The key specifications are single-shot resolution and the “gap-free” or “zero dead-time” capability. The Keysight 53230A universal frequency counter/timer provides 20-ps single-shot resolution, 1 MSa (16 MB) of on-board memory, and gap-free capability that ranges from 1 MSa/s at the high end to one sample per 1000 seconds at the low end (Figure 5). These features enable the 53230A to capture virtually any noise process of interest.

Figure 5. The 53230A includes two 350-MHz input channels and can be configured with a third channel that accepts continuous-wave signals up to 6 GHz (Option 106) or 15 GHz (Option 115).

The 53230A also includes math capabilities with statistics and analysis: mean, standard deviation, maximum, minimum and Allan deviation (square root of the Allan variance). For visual analysis of data, it also provides histograms and trend charts with limit lines.

Using a purpose-built solution

The E5052B SSA implements the more complex phase-detector and cross-correlation methods. It also supports measurements of AM noise and phase noise without changing the RF connection. The E5052B includes low-noise reference sources, an extremely low noise floor, and the DSP capabilities necessary to implement the heterodyne digital discriminator method and the two-channel cross-correlation technique. The SSA is well-suited to measurement offsets as low as 1 Hz and as high as 100 MHz. Its dedicated functionality enables easy operation and simplified setup and calibration.

The modular, PC-based E5500 Series phase noise measurement system can be configured to implement phase-detector techniques such as the reference-source/PLL method or the frequency-discriminator method with an analog delay line. In the reference-source/PLL configuration, the E5500 has the performance and capabilities needed to measure very low phase noise at offsets down to 0.01 Hz when used with a high-performance LO. In frequency-discriminator mode, the system can measure very low phase noise levels at far-out offset frequencies.

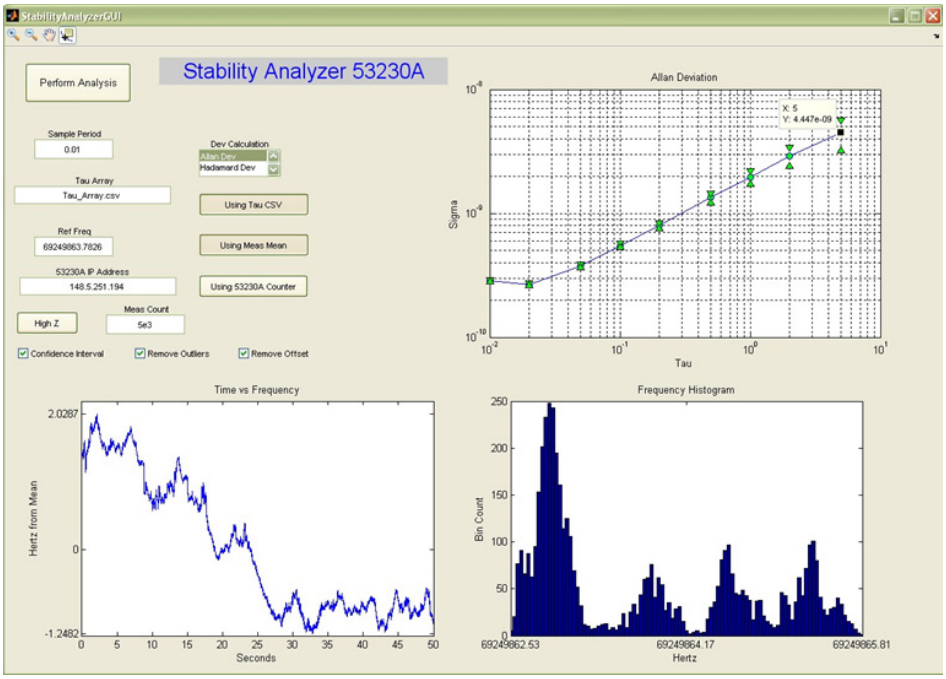
For detailed analysis of clocks, oscillators and other signal sources, The MathWorks offers a free MATLAB program called Stability Analyzer 53230A (2.0)[1]. The program can accept data directly from the 53230A or read it from comma-separated variable (.CSV) files. From that data, the user can characterize stability by choosing the Allan deviation or the Hadamard deviation, which is an adaptation of the Allan deviation similar to the modified Allan deviation. These can be used to produce three types of plots: Allan or Hadamard deviation with optional confidence intervals; frequency versus time; and histogram. All three plot types provide zooming, panning and the ability to select a single point and read its coordinates (e.g., sigma vs. tau, hertz from mean vs. seconds, or bin count versus hertz).

It's worth noting that the Hadamard variance is not affected by linear frequency drift because it uses the second differences of the fractional frequencies. As a result, it can be used for stability analysis of a combination of sources that have different degrees of drift. If there is no drift, the Allan variance and Hadamard variance produce identical results.

Figure 6 presents an example measurement of a 69.521 MHz oscillator. Gap-free measurements were made with a 53230A counter set up to acquire 5,000 measurements with a 10-ms gate time (i.e., total measurement time of 50 seconds). The time-versus-frequency measurement (lower left) reveals two dominant types of noise:

- White FM, which is random noise with a constant spectral density below 10 s and above 30 s.
- Flicker FM, which is noise with a 1/f power density spectrum that yields noise deviations that are inversely proportional to the deviation in hertz from the mean.

In the Allan deviation plot of sigma versus tau (upper right), the slope of the line from 50 ms to 5 s shows that the oscillator signal is being affected by random-walk or Brownian FM noise. This is evidenced by the positive slope in sigma versus time, which corresponds to close-in phase noise in the frequency domain.



Sample Period: 0.01

Tau Array: Tau_Array.csv

Ref Freq: 69249863.7826

53230A IP Address: 148.5.251.194

Meas Count: 5e3

Dev Calculation: Allan Dev / Hadamard Dev

☑ Confidence Interval ☑ Remove Outliers ☑ Remove Offset

Figure 6. MATLAB's Stability Analyzer 53230A calculates statistical measures that provide additional insights for offsets of 10 Hz or lower.

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[1] This requires that you have a licensed copy of MATLAB. The Instrument Control Toolbox is also needed to communicate with the 53230A. The software is available from www.mathworks.com/matlabcentral/fileexchange/.

Converting between domains

To gain deeper familiarity with statistical characterization, frequency-domain results can be converted into the time domain. This is not a trivial process and two methods have proven to be most useful: numerical integration, which is used most commonly, and the power law noise model approximation.

Using numerical integration

Software tools such as MATLAB can be used to make the conversion using numerical integration and thereby produce either the Allan variance or the modified Allan variance. To obtain the desired expressions, we must first relate the time-domain frequency stability to the spectral density of the fractional frequency fluctuations:

$$\sigma^2(\tau) = \int_0^\infty S_y \cdot |H(f)|^2 \cdot df$$

Here, |H(f)|2 is the transfer function of the time-domain sampling function. The two-sample Allan time-domain stability has the following transfer function:

$$|H(f)|^2 = 2\left[\frac{sin^4(\pi\tau f)}{(\pi\tau f)^2}\right]$$

From this, the Allan variance can be calculated as follows:

$$\sigma_y^2(\tau) = 2\int_0^{f_h} S_y(f)\frac{sin^4(\pi\tau f)}{(\pi\tau f)^2}df$$

Taking this one step farther yields the expression for the modified Allan variance:

$$Mod\sigma_y^2(\tau) = \frac{2}{N^4\pi^2\tau_0^2}\int_0^{f_h} S_y(f)\frac{sin^6(\pi\tau f)}{f^2 sin^2(\pi\tau_0 f)}df$$

Using the power law noise models

The domain conversion can be performed through the power law and the following expression:

$$\sigma_y^2(\tau) = h_{-2}\frac{(2\pi)^2}{6}\tau + h_{-1}2\ln 2 + h_0\frac{1}{2\tau} + h_1\frac{1.038 + 3\ln(2\pi f_h\tau)}{2\pi^2\tau^2} + h_2\frac{3f_h}{(2\pi)^2\tau^2}$$

Here, the h_α terms represent the respective levels of the relevant power-law noises as summarized in Table 2.

Noise type	**$\sigma^2_y(\tau)$**	**$S_y(f)$**	***where...***
Random walk (FM)	$A\ f^2\ S_y(f)\ \tau^1$	$A^{-1}\ \tau^{-1}\ \sigma^2_y(\tau)\ f^{-2}$	$A = 4\pi^2/6$
Flicker (FM)	$B\ f^1\ S_y(f)\ \tau^0$	$B^{-1}\ \tau 0\ \sigma^2_y(\tau)\ f^{-1}$	B = 2 ln2
White (FM)	$C\ f^0\ S_y(f)\ \tau^{-1}$	$C^{-1}\ \tau 1\ \sigma^2_y(\tau)\ f^0$	C = ½
Flicker (PM)	$D\ f^{-1}\ S_y(f)\ \tau^{-2}$	$D^{-1}\ \tau^2\ \sigma^2_y(\tau)\ f^1$	$D = 1.038 + 3\ln(2\pi fh\tau 0)/4\pi^2$
White (PM)	$E\ f^{-2}\ S_y(f)\ \tau^{-2}$	$E^{-1}\ \tau^2\ \sigma^2_y(\tau)\ f^2$	$E = 3\ f_h/4\pi^2$

Defining the remaining variables, f_h is the upper cutoff frequency of the measuring system in hertz and τ_0 is the measurement time. These factors apply to only PM flicker noise and PM white noise.

Conclusion

Frequency stability is an important parameter that is crucial to the performance of many modern systems: radar, analog communications and digital communications. The frequency- and time-domain methods described here are complementary techniques that can be combined to achieve deeper insights.

The most efficient and effective method depends on the carrier frequency and the offset frequencies of interest. As a general rule-of-thumb, a signal analyzer equipped with a phase noise measurement application is the best way to get a wide view of signal quality and spectral noise, and also see what's happening at offsets of 20 Hz to 1 MHz. A specialized signal source analyzer such as the E5052B also provides a wide view: it can make accurate measurements at offsets ranging from 1 Hz to 100 MHz. For a closer-in look, a frequency counter with statistical capabilities—and perhaps external software—is effective at offsets of 10 Hz, 1 Hz and 0.1 Hz.

When extremely low residual noise is needed, a dedicated phase noise measurement system is the best choice. The tradeoff is a more complex measurement setup compared to those used with a signal analyzer or universal counter.

Related Information

- Brochure: *X-Series Signal Analysis*, publication 5990-7998EN
- Brochure: *X-Series Measurement Applications*, publication 5989-8019EN
- Technical Overview: *Phase Noise X-Series Measurement Application*, publication 5989-5354EN
- Application Note: *Measurement Considerations for Generating and Analyzing Millimeter-wave Frequencies*, publication 5991-3968EN
- Application Brief: *Exploring Phase Noise Measurement Methods and Techniques*, publication 5991-2069EN
- Data Sheet: *E5052B Signal Source Analyzer*, publication 5989-6388EN
- Data Sheet: *E5500 Series Phase Noise Measurement Solutions*, publication 5989-0851EN
- Selection Guide: *Keysight's Phase Noise Measurement Solutions*, publication 5990-5729EN
- Family Overview: *53200 Series of RF and Universal Frequency Counter/Timers*, publication 5990-6339EN
- Application Note: *Reducing Phase Noise at RF and Microwave Frequencies*, publication 5990-7529EN

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