Keysight Technologies
Understanding Measurement Risk

Application Note

Introduction

Abstract

One key reason for performing a calibration is to assess a device as either in- or out-of-tolerance. Common calibration test scenarios compare a device parameter against that of a measurement standard by way of a measurement process. If the difference between the device parameter and the measurement standard is greater than the specified tolerance, the device is deemed out-of-tolerance. However, errors in the measurement process bring about the possibility of an incorrect assessment. An incorrect assessment may result in devices incorrectly declared as in-tolerance (*false-accept*) or incorrectly declared as out-of-tolerance (*false-reject*).

The risk of making an incorrect in- or out-of-tolerance assessment can be determined by evaluating probability density functions that incorporate a device's parameter population and the measurement error. This paper provides an intuitive explanation of these probability density functions drawing on Monte Carlo simulation to demonstrate the relationship between a device's true value and the corresponding measured value.

In manufacturing facilities throughout the world, test engineers design measurement procedures for manufacturing purposes. It is common for test engineers to rely on the specifications of measuring equipment to assess the accuracy of the measurement procedures. This creates a dependency between the measuring equipment specifications and the quality of the manufacturing process. To maintain manufacturing process quality, the measuring equipment requires periodic calibration.

For the above scenario, one of the primary purposes of calibration is to verify that the measuring equipment performs at a level consistent with the equipment's specifications. In other words, is the measuring equipment in- or out-of-tolerance?

Frequently, calibration involves comparing a device parameter (that is, a parameter of the measuring equipment) against that of a measurement standard. For example, assume we wish to calibrate an RF power source with a power meter. The purpose of the calibration is to assess the RF power source error (the difference between the indicated power and the true power supplied by the source) and determine if it is less than a specified tolerance. If it were possible to use a perfect power meter and a perfect measuring procedure, determining the RF power source's error 2007 NCSL International Workshop and Symposium is simply a matter of noting the difference between the power meter's reading and the indicated value of the RF power source. However, since a real-world power meter is not perfect, knowing the exact RF power source error is not possible. Our lack of knowledge about the exact error is what gives rise to the possibility of declaring a device as in-tolerance when it is actually out-of-tolerance (*false-accept*) or, declaring a device as out-of-tolerance when it is actually in-tolerance (*false-reject*).

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Device Error

From our example, the RF power source error is the value that we wish to compare against the tolerance limit. The RF power source displays the power level it purports to output. The RF power source error is the difference between the purported output power level and the actual true value [7] of the power. Expressed mathematically,

$$e_{dut} = n_{dut} - t \qquad \text{Equation (1)}$$

where:

   e_{dut} = RF power source error, which is the device under test error

   n_{dut} = nominal, or displayed RF power level

   t = true power level

To calibrate the RF power source, the power meter measures the true output power from the RF power source. However, the measured power indicated by the power meter differs from the true value because of measurement error. That is,

$$i_{std} = t + e_{std} \qquad \text{Equation (2)}$$

where:

   i_{std} = power indicated by the power meter

   t = true power level

   e_{std} = power meter measurement error

Combining Equations (1) and (2) yields:

$$n_{dut} - i_{std} = e_{dut} - e_{std} \qquad \text{Equation (3)}$$

The left hand side of Equation (3) represents the observed calibration result. That is, the observed calibration result is the difference between the device under test, as displayed by our RF power source, and the calibration standard as indicated by our power meter. Representing the observed calibration result as y, we can rewrite Equation (3) as

$$y = e_{dut} - e_{std} \qquad \text{Equation (4)}$$

where:

   y = observed calibration result

Equation (4) relates the actual device under test error, e_{dut}, and the measurement error, e_{std}, to the observed calibration result, y. A device is in-tolerance if e_{dut} is less than the tolerance limit. That is, for a tolerance limit, L, a device is in-tolerance if:

$$-L \leq e_{dut} \leq L$$

At the time of calibration, however, the precise value of e_{std} is unknown. Consequently, it is not possible to determine the precise value of e_{dut}. What is known is the observed calibration result, y. Fortunately, if e_{std} is small relative to e_{dut}, the observed calibration result, y, is a reasonable approximation to e_{dut}. Therefore, a device *is declared* in-tolerance if:

$$-L \leq y \leq L$$

Random Variables, Tolerance Testing and Risk

Because at the time of calibration the precise value of the device under test error and the measurement error are unknown, it is necessary to represent e_{dut} and e_{std} as statistical quantities. Particularly, e_{dut} and e_{std} are random variables each characterized by a distribution. Assuming that the device under test belongs to a population, the distribution for e_{dut} represents the different error values for each device in that population. The distribution for the measurement error, e_{std}, is the measurement uncertainty of the measurement process.

Tolerance testing involves using the observed result of the calibration to assess a device as either in- or out-of-tolerance. However, the actual in- or out-of-tolerance status of a device depends upon the actual value of the device under test error. Therefore, there is a chance that the observed result of the calibration leads to an incorrect assessment of the device as being either in- or out-of-tolerance. Risk is the probability of making an incorrect assessment.

Monte Carlo Simulation

We can use Monte Carlo methods to simulate the result of calibrating devices using calibration standards that include errors. As such, Monte Carlo simulation provides a way to visualize the relationship between the device under test error, e_{dut}, and the calibration result, y, when treating the variables in Equation (4) as random variables. Monte Carlo simulation is also useful for identifying incorrect in- or out-of-tolerance assessments as well as determining the risk of making an incorrect assessment.

The Monte Carlo simulation operates as follows. Assuming a distribution for the population to which our device under test belongs, we can draw from it a large number of random samples. Each sample represents a value of the error, e_{dut}, for a device under test. For each of these random samples, we can draw another random sample from a distribution representing the measurement error, e_{std}, and subtract it from the sample for the device under test error to give a value for the calibration result, y.
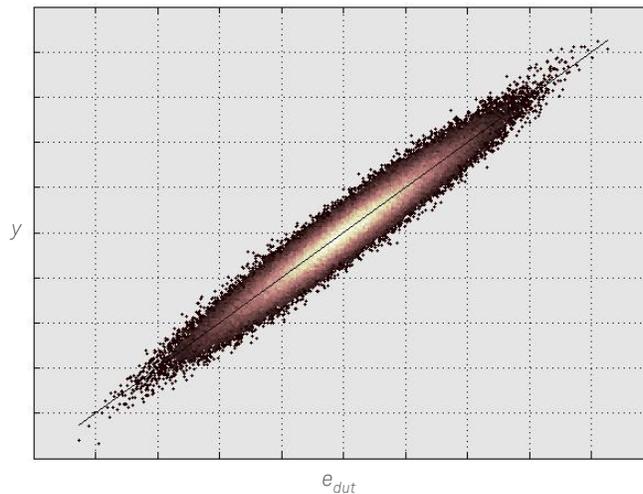

Figure 1. Simulation of observed calibration results versus the device under test error.

The benefit of a Monte Carlo simulation is that values for e_{dut} and the corresponding values for y exist, unlike real-world calibrations where e_{dut} is unknowable. Figure 1 shows values for y plotted against the values for e_{dut} and illustrates graphically that y and e_{dut} are highly correlated. This is necessary in order to approximate a value for e_{dut} using a value for y.

Monte Carlo Simulation (continued)

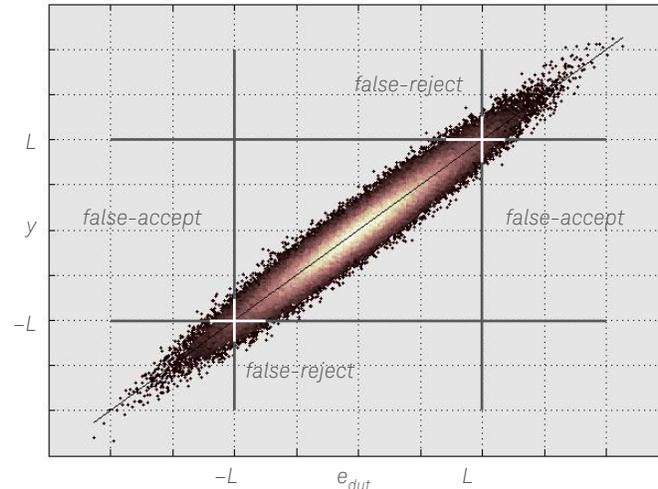Adding tolerance limits to the plot of *y* values versus e_{dut} values defines several different regions.

Figure 2. Tolerance limits defining *false-accept* and false reject regions.

The vertical lines in Figure 2 represent two-sided symmetrical tolerance limits about the device under test error, e_{dut}. Points between the vertical lines represent in-tolerance devices; points outside represent out-of-tolerance devices. The horizontal lines represent two-sided symmetrical tolerance limits about the calibration result, *y*. Points between these lines represent devices *observed* as in-tolerance while points outside represent devices *observed* as out-of-tolerance.

Points in the center region of Figure 2 represent devices that are both truly in-tolerance ($-L \leq e_{dut} \leq L$) as well as observed in-tolerance ($-L \leq y \leq L$). Similarly, points in the outer corners of Figure 2 represent devices that are both truly out-of-tolerance and observed out-of-tolerance. For these two cases, the observed calibration result leads to a correct assessment of a device's actual in- or out-of-tolerance status.

Figure 2 also shows points representing devices observed in-tolerance when, in fact, the device is actually out-of-tolerance. These points lie in the regions labeled *false-accept*. Additionally, Figure 2 shows points representing devices observed as out-of-tolerance when the device's true status is in-tolerance. These points lie in the regions labeled *false-reject*. Points in the false-accept and *false-reject* regions represent devices for which the observed calibration result leads to an incorrect in- or out-of-tolerance assessment.

Risk is the probability of a point lying in either the *false-accept* regions or the *false-reject* regions. With our Monte Carlo simulation, determining risk involves counting the number of points in a particular region and dividing the count by the number of points in the population, or a subset of the population. The divisor, in this case, depends upon the type of risk.

Types of *False-Accept* Risk

Different types of *false-accept* risk exist depending on various views individuals take regarding risk. This section explores three different view cases.

Case A

Case A defines risk as the unconditional probability of observing a device parameter as intolerance when it is actually out-of-tolerance. Case A risk is a quality metric for a calibration laboratory. It describes the likelihood of any given device leaving the calibration laboratory as out-of-tolerance.

Considering our Monte Carlo simulation, Case A risk is the probability of randomly choosing a point in the *false-accept* regions from the entire population. Determining this risk is a matter of counting the points in the *false-accept* regions and dividing by the total number of points in the population.

Case B

Case B defines risk as the conditional probability that a device parameter is out-of-tolerance *given* an in-tolerance observation. Calibration laboratories only put observed in-tolerance devices into service. Case B describes the risk of an observed in-tolerance device that is unknowingly out-of-tolerance. For calibration laboratory management, this risk defines the beginning-of-period measurement reliability for managed calibration equipment.

Determining this risk from our Monte Carlo simulation entails counting the points in the *false-accept* regions and dividing by the number of points where the calibration result is within the tolerance limits ($-L \leq y \leq L$).

Case C

Case C defines risk as the conditional probability that a device parameter is out-of-tolerance *given* a particular observed calibration result. Since a calibration result is particular to a given device, Case C defines risk specific to that device.

For our Monte Carlo simulation, the region of interest is very narrow horizontal slice centered on the given calibration result. Risk, in this case, is the number of out-of-tolerance points divided by the total number of points in this narrow slice.

Joint Probability Density Function

Monte Carlo simulation is useful for visualizing how risk is determined, but a more practical method of determining risk is necessary. This more practical method is available using a joint probability density function appropriate for our calibration scenario. As stated earlier, we wish to treat the variables in Equation (4) as random variables that follow a distribution. We could choose from any number of possible distributions, but based upon the Principle of Maximum Entropy [6], our Monte Carlo simulation assumed that the device population and the measurement error follow Gaussian distributions[1].

The probability density function assumed for the device population is

$$p_0(e_{dut}) = \frac{1}{\sqrt{2\pi}\sigma_0}\, e^{\frac{(e_{dut})^2}{2\sigma_0^2}} \qquad\qquad \text{Equation (5)}$$

where:

> σ_0 = standard deviation of the population distribution

Recalling that the calibration result is the difference between the device under test error and the measurement error, the probability density function assumed for measurement error is a Gaussian distribution centered about a value for e_{dut}. The probability, as a function of y, given a value for e_{dut} is,

$$p_m(y - e_{dut}) = \frac{1}{\sqrt{2\pi}\sigma_m}\, e^{\frac{(y - e_{dut})^2}{2\sigma_m^2}} \qquad\qquad \text{Equation (6)}$$

where:

> σ_m = standard deviation of the measurement error (standard uncertainty)

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1. With better knowledge about the population or the measurement uncertainty, we could choose different distributions or retain the Gaussian distributions, but choose those distributions whose mean value is non-zero.

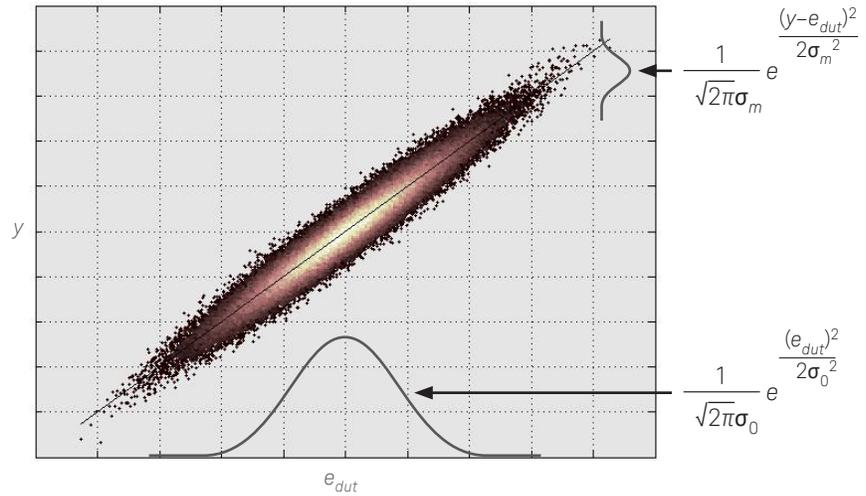# Joint Probability Density Function (continued)

Figure 3. Distribution assumptions for device under test error and measurement error.

Figure 3 illustrates how the device population distribution and the measurement error distribution relate to our Monte Carlo simulation. (It is necessary, however, to imagine the measurement error distribution sliding diagonally across the plot for various values of e_{dut}.) Given the Gaussian assumptions, most points occur near the center of the plot as indicated by the point coloring. The point coloring indicates density with the light coloring representing high point density; dark coloring, low point density[2].

The product of Equations (5) and (6) is the joint probability density function, which defines probability density over a two-dimensional surface area.

$$p(e_{dut}, y) = p_0(e_{dut})\, p_m(y - e_{dut}) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{\frac{(e_{dut})^2}{2\sigma_0^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_m} e^{\frac{(y - e_{dut})^2}{2\sigma_m^2}} \qquad \text{Equation (7)}$$

The total probability for a given two-dimensional rectangular area or region is found by integrating the joint probability density function over the region. That is, the probability for a given region is,

$$P_R = \iint\limits_R p_0(e_{dut})\, p_m(y - e_{dut})\, dA$$

where R defines a particular region on our calibration result versus device under test error plot. We can determine *false-accept* risk by integrating the joint probability density function over the *false-accept* regions of our plot.

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2. For the point coloring, the method to determine density was to draw a grid, count the number of points in each grid square and divide by the grid square area.

Determining *False-Accept* Risk (Case A)

For two-sided tolerance limits, two *false-accept* regions exist (see Figure 4). Case A *false-accept* risk is found by integrating Equation (7) over the *false-accept* regions indicated by the dashed arrows in Figure 4.
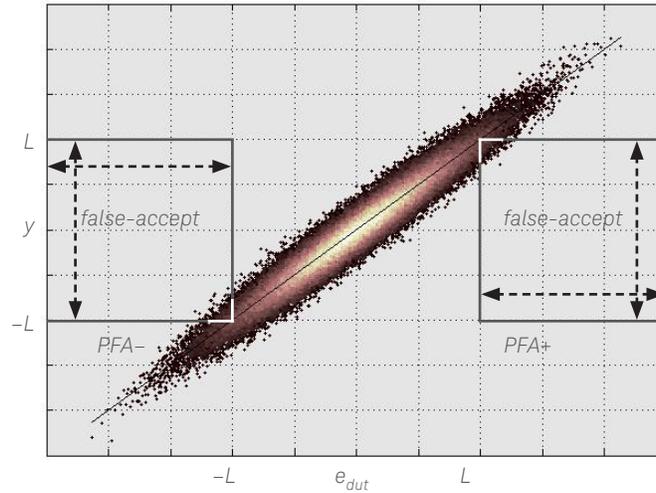

Figure 4. *False-accept* regions for Case A risk.

Specifically,

$$P_{FA^+} = \iint\limits_{FA+} p_0(e_{dut})\, p_m(y - e_{dut})dA$$

where FA^+ is the region:

$$L \le e_{dut} \le \infty \text{ and } -L \le y \le L$$

and

$$P_{FA^-} = \iint\limits_{FA-} p_0(e_{dut})\, p_m(y - e_{dut})dA$$

where FA^- is the region:

$$-\infty \le e_{dut} \le -L \text{ and } -L \le y \le L$$

The total risk is the sum of the probability for each of the two *false-accept* risk regions.

$$P_{Case\,A} = P_{FA^+} + P_{FA^-}$$

Equation (8)

Determining *False-Accept* Risk (Case B)

For Case A risk, integrating Equation (7) provides the unconditional probability for a given region. Case B risk, in contrast, is a conditional probability of *false-accept*. That is, rather than the probability relative to the entire population, Case B risk is the probability of *false-accept* given an observed in-tolerance calibration result ($-L \leq y \leq L$). Conditional probability, in this case, is the (unconditional) probability of a region divided by the (unconditional) probability for a region representing the *condition*. Case B risk is the unconditional probability of *false-accept* divided by the unconditional probability of an observed in-tolerance calibration result.
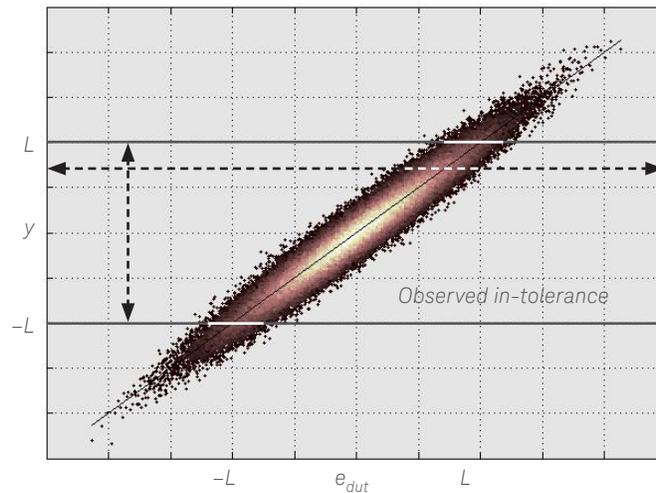

Figure 5. Observed in-tolerance region for Case B risk.

As illustrated in Figure 5, the probability of an observed in-tolerance calibration result is,

$$P_{InTol} = \iint p_0(e_{dut})\, p_m(y - e_{dut})dA \qquad\qquad \text{Equation (9)}$$

where *InTol* is the region:

$$-\infty \leq e_{dut} \leq \infty \text{ and } -L \leq y \leq L$$

The unconditional *false-accept* risk is given by PCase A. Case B risk is,

$$P_{Case\,B} = \frac{P_{Case\,A}}{P_{InTol}} \qquad\qquad \text{Equation (10)}$$

Determining *False-Accept* Risk (Case C)

Case C risk is the conditional probability of *false-accept* given an observed calibration result. It is useful to think of Case C as similar to Case B. Case B only involved values in the region of $(-L \leq y \leq L)$. This region contains the *false-accept* regions and defines the region applied to Equation (9). For Case C risk, this region is infinitely narrow about the observed calibration result. Figure 6 represents this infinitely narrow region as a line cutting horizontally through our calibration result versus device under test error plot.
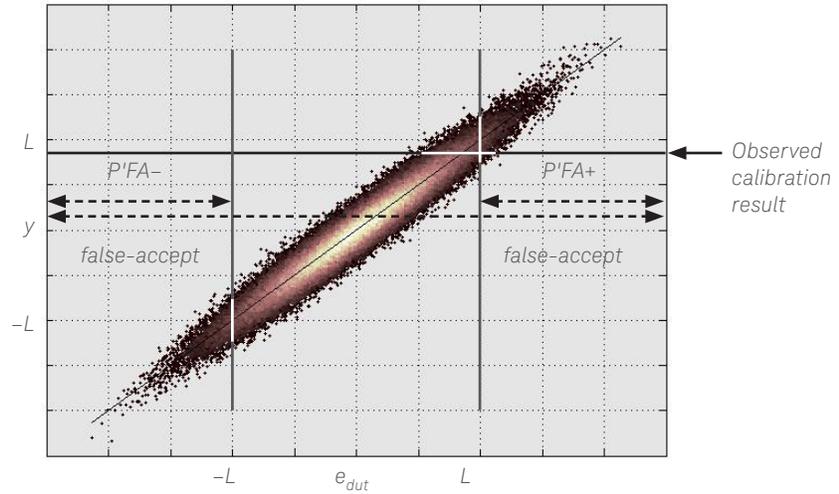

Figure 6. *False-accept* regions for Case C risk.

Case C risk is the probability of *false-accept*, now defined as infinitely narrow horizontal regions positioned at the observed calibration result, divided by the probability of obtaining the observed calibration result. Integrating Equation (7) gives all of these probabilities. Once again, two *false-accept* regions exist. A region exists above the tolerance limit $(L \leq e_{dut} \leq \infty)$ and below the tolerance limit $(-\infty \leq e_{dut} \leq -L)$. Specifically for Case C, a known calibration result exists; therefore, y is constant.

Integrating Equation (7) over the regions indicated by the dashed arrows in Figure 6, assuming a constant value for y, gives,

$$P'_{FA+} = \int_{L}^{\infty} p_0(e_{dut})\, p_m(y - e_{dut}) de_{dut}$$

$$P'_{FA-} = \int_{-\infty}^{-L} p_0(e_{dut})\, p_m(y - e_{dut}) de_{dut}$$

$$P_y = \int_{-\infty}^{\infty} p_0(e_{dut})\, p_m(y - e_{dut}) de_{dut}$$

For Case C, risk is a function of the calibration result.

$$P(y)_{Case\ C} = \frac{P'_{FA+} + P'_{FA-}}{P_y} \qquad \text{Equation (11)}$$

Reducing *False-Accept* Risk

Using a device that is unknowingly out-of-tolerance has financial consequences. Therefore, minimizing *false-accept* risk (as well as *false-reject* risk) is an objective for metrologists.
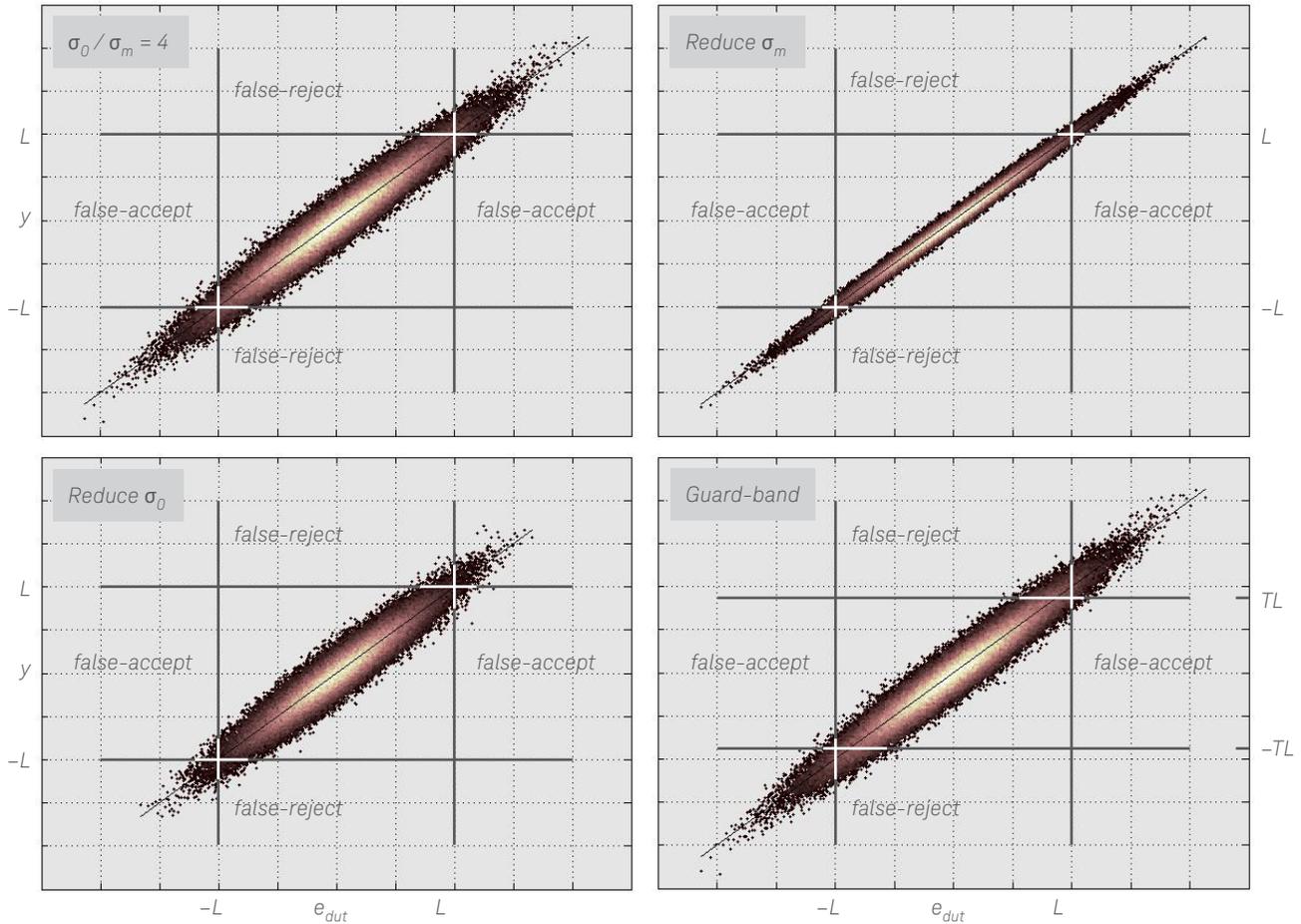

Figure 7. Reducing *false-accept* risk.

Figure 7 illustrates three methods for reducing *false-accept* risk. Each method involves reducing the number of points in the *false-accept* regions. The upper left corner of Figure 7 represents the reference case. Each of the three remain quadrants represent reduced *false-accept* probability from the reference.

Reducing *False-Accept* Risk (continued)

Reduce σ_m

Reducing the standard deviation of e_{std} is to reduce the measurement error. As measurement error lessens, the correlation between the calibration result, *y*, and the device under test error, e_{dut}, improves, making *y* a better approximation to e_{dut}. Likewise, reducing σ_m improves the Test Uncertainty Ratio (TUR) [8].

Represented graphically in the upper right corner of Figure 7, as measurement error lessens, calibration results in the *false-accept* region move vertically to be out-of-tolerance. As well, calibration results in the *false-reject* region move to be in-tolerance. The net result is fewer *false-accept* and *false-reject* points.

Reduce σ_0

Reducing the standard deviation of e_{dut} is to reduce the spread of the device population distribution. This is possible for a calibration laboratory that manages an inventory of devices. If we assume a device experiences random drift (or other mechanisms that cause a device parameter value to change semi-permanently), then the distribution representing the device population tends to spread over time. Managing an inventory of devices with a calibration process that adjusts (or repairs) devices limits the spread of the population[3]. It is possible, therefore, to control σ_0 by selecting appropriate adjustment limits or by adjusting calibration intervals.

The lower left corner of Figure 7 illustrates the effect of reducing the spread of e_{dut}. As e_{dut} narrows, *false-accept* points move horizontally to be in-tolerance, while *false-reject* points move to be out-of-tolerance. Once again, the net result is fewer *false-accept* and *false-reject* points.

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3. Managing devices in this way is generally part of a program to set and manage measurement reliability targets [8] [9].

Reducing *False-Accept* Risk (continued)

Guard-band

Instances arise where reducing *false-accept* risk is necessary and yet, reducing or further reducing e_{std} and e_{dut} is not practical. Applying a guard-band can reduce *false-accept* risk, but at the expense of increased *false-reject* risk.

Guard-banding is to create test limits for the calibration result more restrictive than the tolerance limits. The lower right corner of Figure 7 illustrates this situation where the test limits (indicated as *TL* and *–TL*) are narrower than the tolerance limits.

Guard-banding changes the in- or out-of-tolerance decision point. As test limits becomes more restrictive, more points once considered in-tolerance are now considered out-of-tolerance. The result is fewer *false-accept* points but more *false-reject* points.

It is possible set test limits with sufficient guard-band to achieve desired maximum allowable levels of *false-accept* risk. Because the test limit is different from the tolerance limit, our risk calculations require integration over a modified region. Namely, the unconditional probability for the *false-accept* region above the tolerance limit is, (as a function of the test limit, *TL*),

$$P(TL) = \iint\limits_{GBR} p_0(e_{dut})\, p_m(y - e_{dut})dA \qquad \text{Equation (12)}$$

where *GBR* is the guard-banded region,

$$L \le e_{dut} \le \infty \text{ and } -TL \le y \le TL$$

For Case A and B risk, it is necessary to find a value for TL such that the *false-accept* risk in no greater than the maximum allowable risk.

$$P(TL)_{FA^+} + P(TL)_{FA^-} \le Risk_{max} \qquad \text{(Case A)}$$

$$\frac{P(TL)_{FA^+} + P(TL)_{FA^-}}{P(TL)_{InTol}} \le Risk_{max} \qquad \text{(Case B)}$$

The guard-band is the difference between the test limit that results in the maximum allowable risk and the tolerance.

For Case C risk, it is necessary to find the calibration result for which the *false-accept* risk is no greater than the maximum allowable risk.

$$P(y)_{Case\ C} \le Risk_{max} \qquad \text{(Case C)}$$

For Case C, the difference between the tolerance limit and the calibration result that gives the maximum allowable risk is the guard-band.

Conclusion

Monte Carlo simulation is very useful for visualizing and understanding the subject of measurement risk. This paper presented three similar views of risk. Each depends upon two independent a priori probability distributions: the device under test population and the measurement error distribution. The device under test population, which, by applying the Principle of Maximum Entropy (PME), we determine using equation (5). Referring to the methods of the GUM [10], accounting for all known offset error and continuing with the PME, we determine the measurement error distribution using equation (6). Relying on the GUM, the PME and the joint probability density function provides a very good methodology for determining risk that is appropriate for the purposes of publication and regulation.

For other applications, in addition to publication and regulation, our methodology requires careful understanding of underlying assumptions. For instance, as shown in [5], the computed risk is sensitive to offset of either the device under test population or the measurement error distribution, which we have assumed to be negligible. Nevertheless, the method for determining risk using a joint probability density function is rigorous. The calculated risk value, however, is only as good as the assumptions for both distributions. If desired, we can construct joint probability density functions using distributions that more accurately reflect our knowledge of the device under test population and the measurement error.

Furthermore, we may wish to consider the following:
  – Offset error, at times, is unavoidable. This is especially true for devices that operate over a range of settings or conditions. It is simply not possible to adjust all parameters for all conditions to nominal simultaneously. Equations (5) and (6) are easily adjusted to account for offset.
  – Observed calibration history is useful for estimating device under test distributions. However, care is necessary to ensure the observed data is homogeneous, as indicated in [9]. Several factors that lead to observed multi-mode (i.e., non-homogeneous) distributions relate to device manufacture/model/option, device calibration interval, test station at which the device is tested, as well as functioning devices versus nonfunctioning (broken) devices.
  – The random variables edut and estd from equation (4) are not static over time and therefore, sampling issues need consideration (see [1]).
  – We can extend the joint probability density function to included a weighting function allowing the determination of cost associated with *false-accept*.

$$Cost_R = \iint\limits_R p_0(e_{dut})\, p_m(y - e_{dut})\, w(e_{dut}, y)dA$$

We can choose a weighting function that assigns higher cost values to devices significantly out-of-tolerance compared to devices just out-of-tolerance. Such a costing model requires a clear understanding of the device under test distribution, the measurement error and the weighting function.

Example

Computing risk depends on the tolerance limits, a value for the measurement error standard uncertainty, $\boldsymbol{\sigma}_m$, and an estimate of the device under test population standard deviation, $\boldsymbol{\sigma}_0$. For the purposes of the following example, an estimate[4] for $\boldsymbol{\sigma}_0$ is,

$$\sigma_0 = \frac{L}{F^{-1}\left(\frac{1+p}{2}\right)}$$

Equation (13)

where F^{-1} is the inverse normal function, and p is the in-tolerance probability which is based on the observed history for a population of devices.

In this example, the tolerance limits for the output power accuracy of an RF power source are ± 0.9 dB. A power meter with a 95% expanded uncertainty of ± 0.274 dB is used to calibrate the RF power source. From the calibration history, we will assume that the in-tolerance probability is 80%. We wish to establish guard-bands to limit the *false-accept* risk to 2%.

From the above,

$$L = 0.9$$

$$\sigma_m = \frac{0.274}{1.96} = 0.14$$

$$p = 0.8$$

$$\sigma_0 = \frac{0.9}{F^{-1}\left(\frac{1+0.8}{2}\right)} = 0.7$$

With values for L, $\boldsymbol{\sigma}_0$, $\boldsymbol{\sigma}_m$, we can use Equations (8) and (10) to determine Case A and B *false-accept* risk.

$$P_{FA^+} = \int_{0.9}^{\infty} \int_{-0.9}^{0.9} p(e_{dut})\, p(y - e_{dut}) dy de_{dut}$$

$$P_{FA^-} = \int_{-\infty}^{-0.9} \int_{-0.9}^{0.9} p(e_{dut})\, p(y - e_{dut}) dy de_{dut}$$

$$P_{Case\,A} = P_{FA^+} + P_{FA^-} = 2.370\%$$

$$P_{InTol} = \int_{-\infty}^{\infty} \int_{-0.9}^{0.9} p(e_{dut})\, p(y - e_{dut}) dy de_{dut}$$

$$P_{Case\,B} = \frac{P_{Case\,A}}{P_{InTol}} = 2.996\%$$

4. The use of Equation (13) is by convention. Case C risk and guard-band can be computed using the software tool RiskGuard [2] [3] provided by Integrated Sciences Group, which estimates σ_0 using Equation (13). Similar methods for estimating σ_0 are found in [4].

Example (continued)

This example does not provide a calibration result. For that reason, Case C risk is not determined.

Both the Case A and Case B *false-accept* risks are greater than the desired 2%. The goal is to find a test limit, employing Equation (12), that achieves the desired *false-accept* risk. Doing so is an iterative process for which a numerical solver[5] is most efficient.

$$TL_{Case\ A} = 0.881\ dB$$
$$TL_{Case\ B} = 0.853\ dB$$

The guard-band in these cases is the difference between the test limit and the tolerance limit.

$$GB_{Case\ A} = 0.9 - 0.881 = 0.019\ dB$$
$$GB_{Case\ B} = 0.9 - 0.853 = 0.047\ dB$$

To determine Case C guard-band, it is necessary to find the calibration result, y, using Equation (11), that achieves the desired *false-accept* risk. Doing so is once again an iterative process best executed using a numerical solver.

$$y_{Case\ C} = 0.643\ dB$$
$$GB_{Case\ C} = 0.9 - 0.643 = 0.257\ dB$$

Table 1 summarizes the example.

Table 1 Example Summary.

	Case A	Case B	Case C
	Unconditional probability of *false-accept*	**Probability of *false-accept* given observed in-tolerance**	**Probability of *false-accept* given a measured value**
Probability of *false-accept*	2.370 %	2.996 %	NA
2% risk guard-band	0.019 dB	0.047 dB	0.257 dB

5.   For this example, the *fsolve* function from the MATLAB Optimization Toolbox was used.

References

[1]   Brown, R., *Using a Manufacturer's Specification as a Type B Error Contribution*, NCSL International Workshop and Symposium, Nashville, TN, August, 2006

[2]   Castrup, H., *Estimating Guardbands Commensurate with Allowable Risk*, August 18, 2000

[3]   Castrup, H., *Analytical Metrology SPC Methods*, Proc. 1991 NCSL Workshop and Symposium, August 1991, Albuquerque, NM

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