Keysight Technologies
High Attenuation Measurement of Step Attenuators

Application Note

Abstract

This paper introduces a solution for high attenuation measurement of step attenuators. Fundamentally, this high attention measurement method is based on the cascaded 2-port network and S-parameter theory; this method is to compute S-parameters of high attenuation (> 80 dB) using the measured S-parameters of lower attenuation (≤ 80 dB) settings, and the calculations depend on attenuator card sequence and physical structure of step attenuator. This method can measure the attenuation high as 120 dB.

This is NOT a straight dB addition, this solution can offer considerable accuracy only using VNA (Vector Network Analyzer), and it uses T-matrix (as known as transmission parameter or cascade parameter) method which can make the calculations easier. Measurement uncertainties are derived from uncertainties of cascaded S-parameters, for example, measurement uncertainty for 80 dB at 18 GHz is less than 0.8 dB and 110 dB at 18 GHz is less than 1.0 dB.

KEYSIGHT
TECHNOLOGIES
Unlocking Measurement Insights

Introduction

There was a need to verify the accuracy of an attenuator in a new synthesizer product. This method provides a simpler procedure for Calibration Lab using an automatic measurement system to perform high attenuation measurement of step attenuators. This T-matrix method was originally suggested by the project manager, and finally implemented by software engineer. The author of this report, as metrologist of the project, provided principle verification, experimentation results review and measurement uncertainty analysis. This method was also approved by expert from Keysight Technologies Component Test Division.

This measurement system has been used to calibrate a large number of step attenuators for many years. This paper describes the T-matrix measurement method for achieving high accuracy, and will introduce details of using cascade parameters to represent each thru-line and attenuator section based on attenuator physical structure of step attenuator.

T-matrix Description

The following discussion in general applies to a cascade of N-port networks. For the sake of simplicity, however, we limited our analysis to two-port networks only. When cascading a number of two-port network in series, a more useful network representation is needed to facilitate the calculation of the overall network parameters.

This representation should relate the output quantities in terms of input quantities. Using such a representation will enable us to obtain a description of the completed cascade by simply multiplying together the matrix describing each network.

The following information on 2-port network is available from an Keysight application note; see reference 2 at the end of this paper. 2-port network (Figure 1) can be used to model many components, and attenuator is a typical example. The 2-port network can be characterized by S parameter matrix (Figure 2). For 2-port networks the S -parameters are defined as:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

The inputs and outputs of the 2-port network can be denoted as:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Where S_{11} is the input reflection coefficient with the output port terminated by a matched load ($a_2 = 0$).

Figure 1. 2-port network

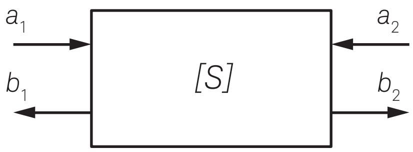


Figure 2. S-parameters for 2-port network

T-matrix Description (Continued)

Therefore:

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0}$$

Similarly, S_{21} is the forward transmission coefficient indicating with the output port terminated by a matched load (a_2 =0):

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0}$$

S_{22} is the output reflection coefficient with the input terminated by a matched load (a_1 = 0):

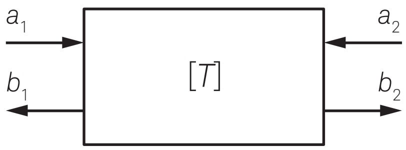
$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_2 = 0}$$

S_{12} is the reverse transmission coefficient with the input terminated by a matched load (a_1 =0)

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0}$$

Transmission matrix [T] is expressed in terms of the waves at the input port and the waves at the output port. Using this definition the transmission matrix formulation becomes very useful when dealing with multistage circuits or infinitely long periodic structures such as those used in circuits for traveling wave tubes, etc.

The transmission matrix for a two-port network, as shown in Figure 3, is defined as:



$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

Figure 3. T-parameters for 2-port network

T-matrix Description (Continued)

The relationship between S- and T- parameters can be derived using the above basic definition as follows:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} -\dfrac{S_{11} \cdot S_{22} - S_{12} \cdot S_{21}}{S_{21}} & \dfrac{S_{11}}{S_{21}} \\ -\dfrac{S_{22}}{S_{21}} & \dfrac{1}{S_{21}} \end{bmatrix}$$

The reverse relationship expressing [S] in terms of [T] matrix can also be derived with the following result:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{T_{12}}{T_{22}} & \dfrac{T_{11} \cdot T_{22} - T_{12} \cdot T_{21}}{T_{22}} \\ \dfrac{1}{T_{22}} & -\dfrac{T_{21}}{T_{22}} \end{bmatrix}$$

For a cascade connection of two-port networks, as shown in Figure 4, the overall T-matrix can be obtained as follows:

$$T = T_1 * T_2$$

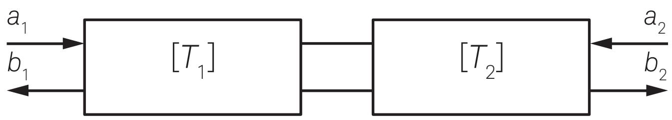


Figure 4. T-parameters for 2 cascaded networks

Thus, the total *T*-matrix is the multiplication of the two T-matrices. This is the theoretical basis for step attenuator measurement.

T-Matrix Method for Step Attenuator Calibration

The description on calibration method with T-matrix method will use a Keysight 8496B step attenuator as an example. Step attenuator sections are connected in cascade. Each section consists of a precision, thin-film attenuator card, a lossless thru-line and a ganged pair of edge line transmission lines. The edge lines are flexed to make contact with either the attenuator card or the thru-line. The edge line contacts are gold-plated leaf springs which ensure long life and high repeatability.

Table 1 shows the attenuator switching order. Figure 5 shows the attenuator card sequence and physical structure of step attenuator 8496B.

Table 1. Attenuator switching order

8496A/B attenuator sections				
Attenuator (dB)	**1 10 dB**	**2 20 dB**	**3 40 dB**	**4 40 dB**
0				
10	X			
20		X		
30	X	X		
40				X
50	X		X	
60		X	X	
70	X	X	X	
80			X	X
90	X		X	X
100		X	X	X
110	X	X	X	X

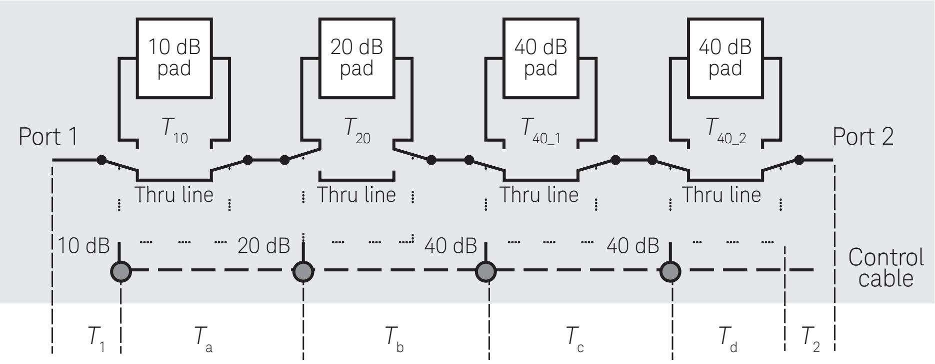


Figure 5. 8496B individual pads connection

Below are 110 dB attenuation calculations, using this Keysight 8496B step attenuator as an example.

$$T_{0\_m} = T_1 * T_a * T_b * T_c * T_d * T_2 \rightarrow T_{0\_m}^{-1} = T_2^{-1} * T_d^{-1} * T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1}$$
$$T_{10\_m} = T_1 * T_{10} * T_b * T_c * T_d * T_2 \rightarrow T_{10} = T_1^{-1} * T_{10\_m} * T_2^{-1} * T_d^{-1} * T_c^{-1} * T_b^{-1}$$
$$T_{20\_m} = T_1 * T_a * T_{20} * T_c * T_d * T_2 \rightarrow T_{20} = T_a^{-1} * T_1^{-1} * T_{20\_m} * T_2^{-1} * T_d^{-1} * T_c^{-1}$$
$$T_{40\_1\_m} = T_1 * T_a * T_b * T_{40\_1} * T_d * T_2 \rightarrow T_{40\_1} = T_b^{-1} * T_a^{-1} * T_1^{-1} * T_{40\_m\_1} * T_2^{-1} * T_d^{-1}$$
$$T_{40\_2\_m} = T_1 * T_a * T_b * T_c * T_{40\_2} * T_2 \rightarrow T_{40\_2} = T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1} * T_{40\_2\_m} * T_2^{-1}$$

T-Matrix Method for Step Attenuator Calibration (Continued)

Combine above parameters to get attenuation 110 dB:

$$
\begin{aligned}
T_{110\_m} &= T_1 * T_{10} * T_{20} * T_{40\_1} * T_{40\_2} * T_2 \\
&= T_1 * (T_1^{-1} * T_{10\_m} * T_2^{-1} * T_d^{-1} * T_c^{-1} * T_b^{-1}) * (T_a^{-1} * T_1^{-1} * T_{20\_m} * T_2^{-1} * T_d^{-1} * T_c^{-1}) * (T_b^{-1} * T_a^{-1} * T_1^{-1} * T_{40\_1\_m} * \\
&\quad T_2^{-1} * T_d^{-1}) * (T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1} * T_{40\_2\_m} * T_2^{-1}) * T_2 \\
&= T_{10\_m} * (T_2^{-1} * T_d^{-1} * T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1}) * T_{20\_m} * (T_2^{-1} * T_d^{-1} * T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1}) * T_{40\_1\_m} * (T_2^{-1} * T_d^{-1} \\
&\quad * T_c^{-1} * T_b^{-1} * T_a^{-1} * T_1^{-1} *) * T_{40\_2\_m} \\
&= T_{10\_m} * T_{0\_m}^{-1} * T_{20\_m} * T_{0\_m}^{-1} * T_{40\_1\_m} * T_{0\_m}^{-1} * T_{40\_2\_m}
\end{aligned}
$$

Note: T_{0_m}, T_{10_m}, T_{20_m}, $T_{40\_1\_m}$, $T_{40\_2\_m}$ and T_{110_m} can be measured directly.

Comparing the results of directly measurement and T-matrix measurement can get below data as shown as Table 2.

- Measurement condition:
- DUT: 8496B
- ETE: Keysight PNA and 8902A/PSA.
- Test frequency: 1 GHz, 18 GHz

Note: The results of "T-matrix measurement" are measured with network analyzer PNA and calculated to get the results when attenuation is larger than 40 dB. The results of "Direct measurement" are measured with 8902A measuring receiver and spectrum analyzer manually, and all results are measured directly.

This method has been used for signal integrity analysis. For technique details refer to the references 3 and 4 at the end of this paper.

Table 2. Validation results

Frequency = 1 GHz	**Attenuation measurement results (dB)**			
Attenuation	**T-matrix measurement**	**Direct measurement**	**Difference**	**Measurement uncertainty**
0	0.17	0.16	0.01	0.05
10	10.04	10.05	-0.01	0.06
20	19.93	19.92	0.01	0.07
30	30.01	29.98	0.03	0.08
40	40.06	40.03	0.03	0.09
50	50.1	50.09	0.01	0.12
60	59.99	59.98	0.01	0.19
70	70.03	70.03	0.00	0.16
80	80.12	80.07	0.05	0.11
90	90.16	90.12	0.04	0.11
100	100.05	100.01	0.04	0.12
110	110.09	110.06	0.03	0.14

Frequency = 18 GHz	**Attenuation measurement results (dB)**			
Attenuation	**T-matrix measurement**	**Direct measurement**	**Difference**	**Measurement uncertainty**
0	1.11	1.24	–0.13	0.12
10	10.03	9.99	0.04	0.15
20	20.09	20.09	0	0.15
30	30.22	30.22	0	0.16
40	40.32	40.27	0.05	0.19
50	50.37	50.34	0.03	0.25
60	60.52	60.47	0.05	0.3
70	70.54	70.48	0.06	0.37
80	80.64	80.57	0.07	0.38
90	90.68	90.96	–0.28	0.38

Uncertainty Analysis

This measurement uncertainty analysis is for all step attenuators (DUT) Transmission (S21, S12) calibration by using a network analyzer. The information is based on the network analyzer specified frequency range (options are considered due to DUT frequency range requirement). The raw measurement uncertainty analysis data was derived from the Keysight announced tool (Uncertainty Test revision spread sheet) A.2.6.1, DLL Revision 4.7.0.8 for the attenuation below or equal to 65 dB attenuations. For high attenuation measurement uncertainty analysis, can refer to below arithmetic to drive its uncertainty using the uncertainties of 10, 20, 30 and 40 dB, based on the measurement methodology (T-matrix) at the description section above. Table 3 shows the calculated measurement uncertainties based on the attenuation settings being measured.

Table 3. Measurement uncertainty

Frequency (GHz)	Measurement uncertainty (dB)				
	10 dB	**20 dB**	**30 dB**	**40 dB**	**50 dB**
1	0.055	0.065	0.076	0.089	0.122
2	0.055	0.065	0.077	0.090	0.128
2	0.090	0.099	0.109	0.123	0.148
3	0.094	0.103	0.113	0.129	0.160
4	0.099	0.107	0.116	0.135	0.171
5	0.103	0.111	0.120	0.140	0.181
6	0.107	0.115	0.124	0.145	0.190
7	0.111	0.118	0.127	0.150	0.198
8	0.114	0.122	0.130	0.154	0.206
9	0.118	0.125	0.134	0.158	0.213
10	0.122	0.128	0.137	0.162	0.220
11	0.125	0.131	0.140	0.166	0.226
12	0.128	0.135	0.143	0.170	0.231
13	0.131	0.138	0.146	0.173	0.236
14	0.135	0.141	0.149	0.177	0.240
15	0.138	0.144	0.152	0.180	0.244
16	0.141	0.147	0.155	0.183	0.248
17	0.144	0.150	0.158	0.186	0.251
18	0.147	0.153	0.161	0.189	0.255
19	0.150	0.156	0.164	0.192	0.257
20	0.153	0.159	0.168	0.195	0.260
21	0.156	0.163	0.171	0.198	0.263
22	0.159	0.166	0.174	0.202	0.265
23	0.163	0.170	0.178	0.205	0.268
24	0.166	0.173	0.181	0.208	0.270
25	0.169	0.177	0.185	0.211	0.272
26	0.173	0.181	0.189	0.215	0.275
26.5	0.175	0.183	0.191	0.216	0.276

Uncertainty Analysis (Continued)

As an example, Figure 6 indicates a connection of two attenuation sections, the measurement uncertainty of attenuation (A+B) dB can be derived as below formula:

$$S_{11} = f_{11}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru})$$
$$S_{12} = f_{12}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru})$$
$$S_{21} = f_{21}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru})$$
$$S_{22} = f_{22}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru})$$

$$\begin{aligned}\Delta S_{11\_1} = (&f_{11}\,(S_{11}^{A} + \Delta S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru})\\ &- f_{11}\,(S_{11}^{A} - \Delta S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru}))/2\end{aligned}$$

$$\vdots$$

$$\begin{aligned}\Delta S_{11\_12} = (&f_{11}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru} + \Delta S_{22}^{thru})\\ &- f_{11}\,(S_{11}^{A}, S_{12}^{A}, S_{21}^{A}, S_{22}^{A}, S_{11}^{B}, S_{12}^{B}, S_{21}^{B}, S_{22}^{B}, S_{11}^{thru}, S_{12}^{thru}, S_{22}^{thru} - \Delta S_{22}^{thru}))/2\end{aligned}$$

$$S_{11\_type\_B} = SQRT\,((\Delta S_{11\_1})^2 + (\Delta S_{11\_2})^2 + \ldots. + (\Delta S_{11\_12})^2)$$
$$S_{11\_MU} = K\_factor * SQRT\,((\Delta S_{11\_type\_B})^2 + (\Delta S_{11\_type\_A})^2)$$

This analysis is also used to get measurement uncertainty for high attenuation with pads connection.

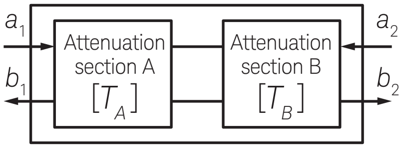


Figure 6. Two individual pads connection

Conclusion

This method considers mismatch impact between each pad and thru lines inside the step attenuator using cascaded T-parameter; this system has been used to calibrate a large number of step attenuators with impressive results, the actual measurement results and measurement uncertainties show this method can archive the same measurement accuracy as the direct measurement, but can reduce calibrate time for Cal Labs, it has also been a valuable tool for characterization of other fix attenuators with high attenuation.

References

1. Matthew M. Radmanesh, Radio Frequency and Microwave Electronics Illustrated.
2. Hewlett Packard, Co., Application Note 95-1, S-parameter techniques.
3. Keysight Technologies, Signal Integrity Analysis Series Part 3: The ABCs of De-embedding, Application Note, literature number 5989-5765EN
4. Keysight Technologies, De-embedding and Embedding S-parameter Networks Using a Vector Network Analyzer, Application Note, literature number 5980-2784EN

Acknowledgement

The author wishes to express her gratitude to people from Keysight Technologies. Ken Wong (Expert from Component Test Division) and Dahai Sun (Manager from WCSS) for investigation support and Yu Gu (Senior software engineer from WCSS) for operating and software implementation.

Published in USA, May 20, 2015
5991-1268EN
www.keysight.com