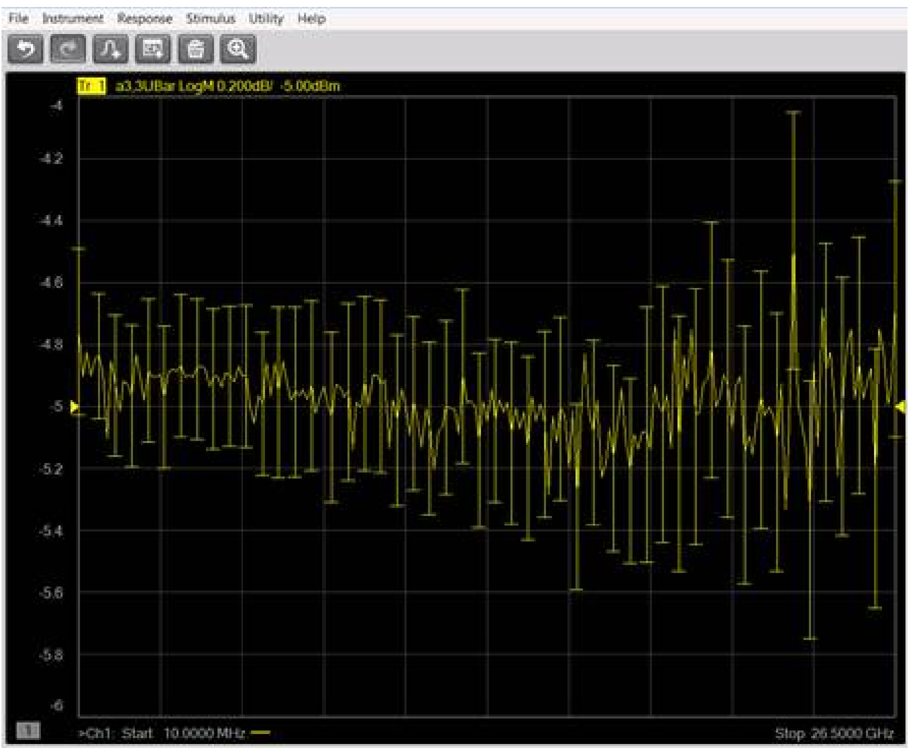
APPLICATION NOTE

VNA Measurement Uncertainty

Real Time S-Parameter and Power Uncertainty

The vector network analyzer (VNAs) provides the most accurate S-parameter and power measurements for the microwave industry today. However, the computation of measurement uncertainty under actual measurement conditions is both challenging and cumbersome. This application note will show how to use Keysight's innovative real time uncertainty VNA software option (S93015B) to determine the uncertainty of your S-parameter and power measurements.



Introduction

Keysight Technologies has developed a innovative technology that allows for the most complete accuracy estimation in real-time. This technology allows users of the PNA and PNA-X family of VNA's to get the measured data and accuracy information at the same time. All of this is done without compromising the measurement speed. Furthermore, the approach is seamlessly integrated into the standard PNA / PNA-X measurement and calibration processes and requires little additional user effort.

The real-time uncertainty information provided by the instrument allow users to:

- Have greater confidence in the reproducibility of their measurements avoiding time consuming repeated testing.
- Implement pass/fail tests easily because now the instrument quantifies the gray region that is in between a full pass or a full fail.
- Apply more realistic limit-lines which can increase the production yield and reduce the defect percentage on the finished products.
- Easily establish a metric to quantify the quality of the measurement process, so your company's quality control procedures are simplified.
- Automatically include the uncertainty information for most Keysight calibration kits, an industry-first service.
- Get traceability through Keysight's calibration kits, which are traceable to various National Metrology Institutes.
- Include uncertainty information to your product specifications and datasheet.

This application covers the following:

- A brief review of uncertainty concept applied to complex numbers.
- An introduction to the new VNA uncertainty model.
- The procedure to follow for the new option on a PNA-X.
- An example of derived quantities' uncertainty by using the new data file available from the S93015B software option.

Uncertainty Fundamentals

Often, the terms “errors” and “uncertainties” are confused when talking about the accuracy of a certain measurement process.

When referring to vector network analyzers’ measurements, “error” denotes a systematic effect that has a direct effect on the accuracy of a measurement, causing it to be different from the *true value* of the parameter. The “errors” are repeatable and can be measured using proper “calibration” procedures. While “uncertainties” do degrade the accuracy as well, they express the level of *doubt* on the measurement result. As such they become unique to a specific measurement configuration.
There are ways, though, to estimate how much “uncertainties” affect the accuracy of a measurement; for example, the measurement noise is properly characterized performing many repeated measurements and then applying statistical analysis on the collected data. Other uncertainty sources, like imperfect calibration standards’ definitions, can be characterized by other means, often requiring much effort [1,2].
One of the difficulties of expressing and understanding uncertainties with Scattering parameters is that vector network analyzers measure complex quantities having real and imaginary parts, or from a different point of view, magnitude and phase. As example suppose the task is to perform many repeated measurements on the same device, each time calibrating the VNA, connecting the DUT and taking the data. After collecting all measurements, the result will be similar to the simulation in Figure 1, which plots the real/imaginary measurement values as function of the measurement (sample) number.

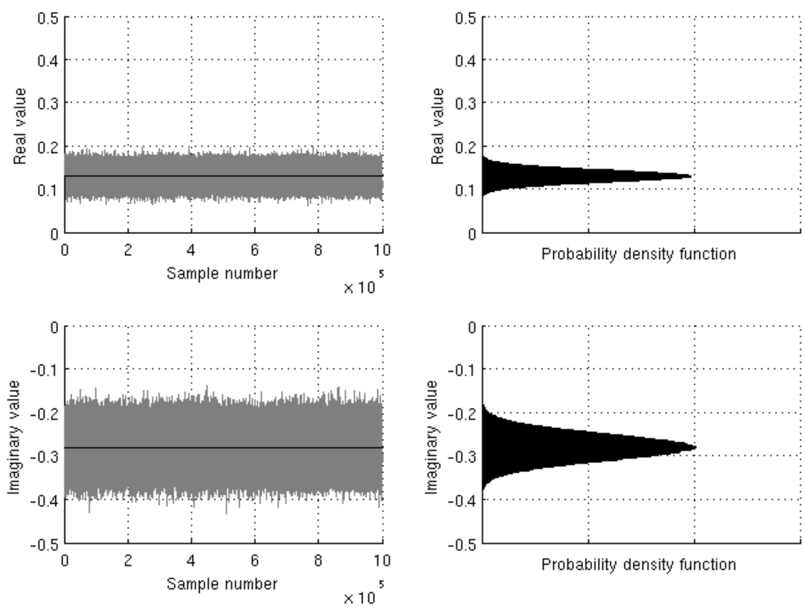


Figure 1: An example of repeated complex data measurements.

The plots on the right are histograms and show the probability density of samples in fine intervals around the real and imaginary mean values. The shape of those histograms closely resembles a gaussian curve, which has a peak at the mean value, and symmetrically decreases for values different from the mean ones.

If the data are represented in a polar plot or Smith Chart, we could find something similar to Figure 2, where the data points are now scattered around the mean value in a elliptical region.

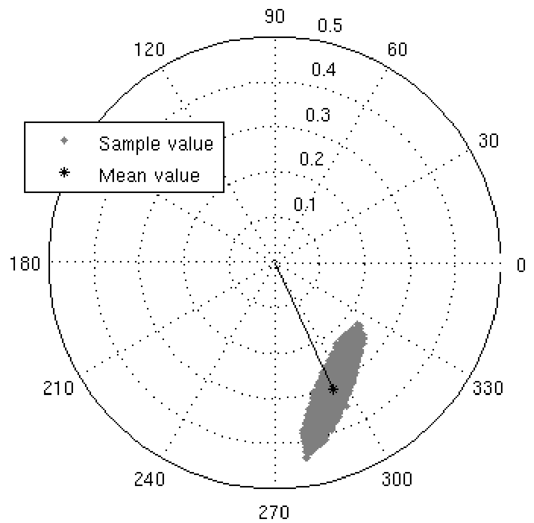


Figure 2: A polar view of repeated complex data measurements.

The shape and orientation of this ellipse depends on how the real and imaginary uncertainties interact with each other, as will be shown in the following section.

It is also interesting to plot the phase and magnitude distributions of our measurement data on a polar plot. Figure 3 represents a phase histogram, which counts the number of measurements having phase in fine intervals around the phase of the mean value; the plot is somewhat similar to an antenna radiation pattern: the peak corresponds to the phase of the majority of the samples, while the “3 dB points” can represent the phase spread. For convenience, the right part shows the same information as cartesian histogram.

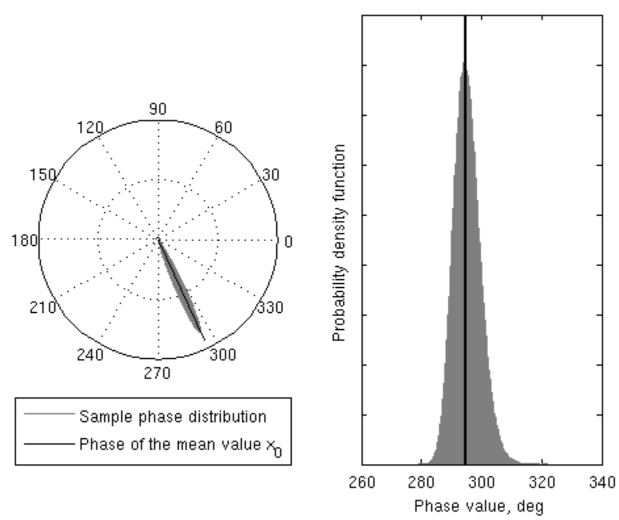


Figure 3: Phase histogram of repeated complex data measurements.

Figure 4 represents the magnitude histogram, which counts the number of measurements having magnitude in certain intervals. In both the magnitude and phase histograms, the peak is close to the magnitude or phase of the mean value; however, that is not always true, and in general the mean magnitude and mean phase *do not* correspond to the mean values of magnitude and phase. Note that the histograms are not symmetrical nor Gaussian-shaped.

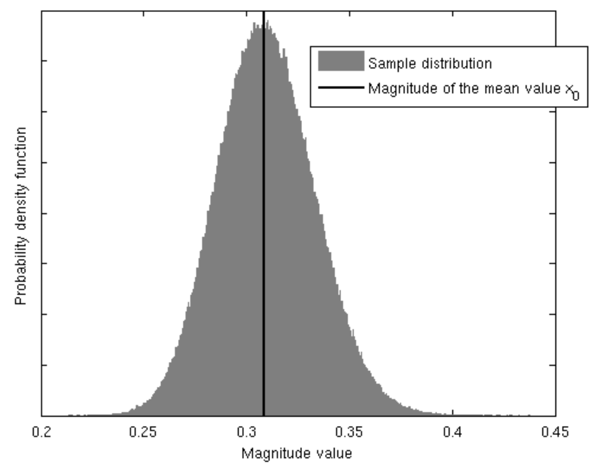


Figure 4: Magnitude histogram of repeated complex data measurements.

Complex Quantities Uncertainty representation

As we have seen, repeated S-parameter measurements tend to scatter in ellipse-shaped regions. Measurements very close to the mean value are much more frequent and become rare when the distance from the mean value is increased. This opens up to the concept of *confidence regions*, also known as *uncertainty ellipses* centered on the mean value which progressively scale in dimensions while maintaining the same principal axis ratio and orientation. Such ellipses define “probability borders”, that is, there is a certain probability that the next measurement will fall inside the ellipse, as shown in Figure 5.

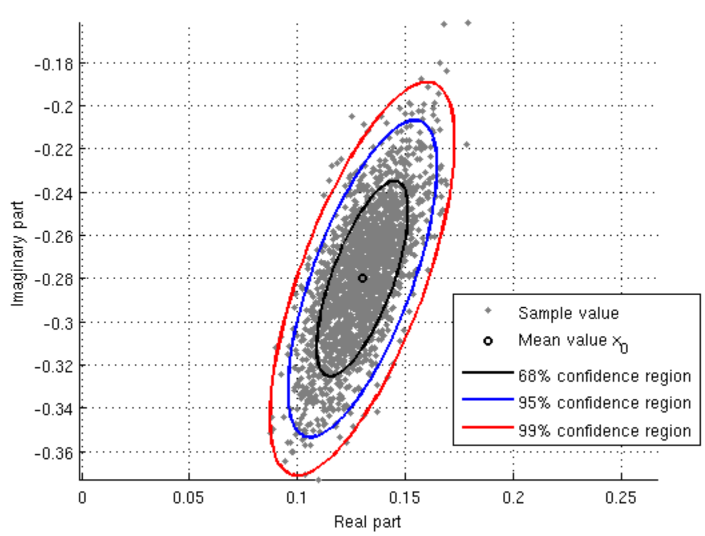


Figure 5: Confidence regions of repeated complex data measurements.

Clearly, evaluating the measurement uncertainty by repeating the same measurement tens or hundreds of times the same measurement is not practical in many situations. However, the uncertainty of any single measurement will have the same statistical properties as any other measurement taken under the same conditions. Knowledge of the statistical properties can be used to plot the elliptical the confidence regions on the Smith Chart, and from these it is possible to compute the real, imaginary or magnitude and phase uncertainties. Numerically, the uncertainty of a complex number is stored as a 2×2 *covariance matrix*, from which it is possible to compute the ellipse shape and rotation information.

Given a complex number, with its real and imaginary parts

$$x = x_r + jx_i \qquad (1)$$

The corresponding covariance matrix of *x* consists of four terms:

$$V_x = \begin{bmatrix} V_{rr} & V_{ri} \\ V_{ir} & V_{ii} \end{bmatrix} \qquad (2)$$

where V_{rr} is the variance of the real part, V_{ii} the variance of the imaginary part, and the other two terms, V_{ri} and V_{ir}, express the co-variance between the real and imaginary parts; in this case, the two terms are equal, that is $V_{ri} = V_{ir}$, making the covariance matrix symmetrical.

While the variances can be regarded as the "power" of the real or imaginary uncertainties, the covariances provide further information on the dependence of one part to the other, which is easier to understand if the correlation factor is introduced:

$$\varrho_x = \frac{V_{ri}}{\sqrt{V_{rr} V_{ii}}} \qquad (3)$$

when the correlation factor is zero, the real and imaginary uncertainties are independent; when it is 1 or −1 (it can be negative) the two components are fully dependent.

The shape and rotation of the error ellipse can be computed from the covariance matrix (see Figure 6);

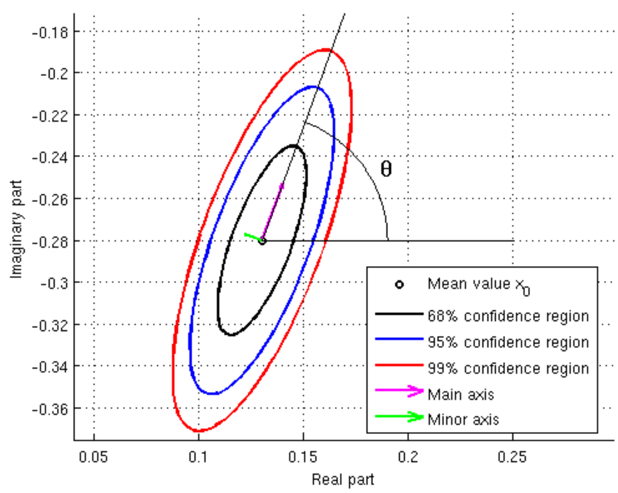


Figure 6: Confidence regions, axis and orientation

the axis lengths are proportional to the square root of the covariance matrix eigenvalues λ_1 and λ_2:

$$l_1 = \mu\sqrt{\lambda_1} \quad \rightarrow \quad \text{main axis} \qquad (4)$$

$$l_2=\mu\sqrt{\lambda_2} \quad \rightarrow \quad \text{minor axis} \tag{5}$$

where μ is a multiplying constant based on the chosen confidence level for the error ellipse (typically 68%, 95% and 99%), while the eigenvalues are:

$$\lambda_1 = \frac{V_{rr}+V_{ii}+\sqrt{(V_{rr}-V_{ii})^2+4V_{ri}^2}}{2} \tag{6}$$

$$\lambda_2 = \frac{V_{rr}+V_{ii}-\sqrt{(V_{rr}-V_{ii})^2+4V_{ri}^2}}{2} \tag{7}$$

And the main axis is rotated with respect to the real axis by an angle:

$$\Theta=\arctan\left(\frac{2V_{ri}}{V_{rr}+-V_{ii}+\sqrt{(V_{rr}-V_{ii})^2+4V_{ri}^2}}\right) \tag{8}$$

From the covariance matrix, it is possible to compute the magnitude and phase uncertainties as well as the real and imaginary uncertainties. The graphic effect is shown in Figure 7 and Figure 8.

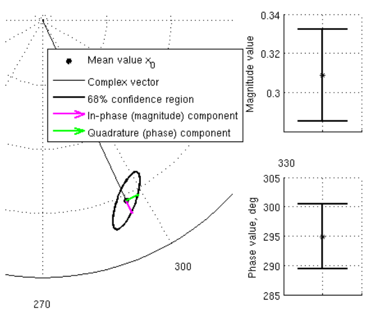


Figure 7: Magnitude and phase uncertainties.

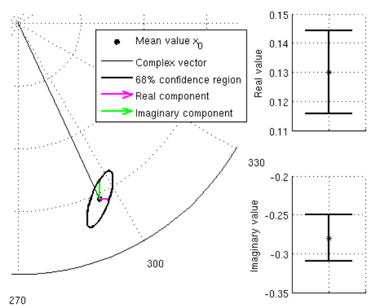


Figure 8: Real and imaginary uncertainties.

When plotting the magnitude uncertainty in decibel scale, one must be aware that the upper and lower bounds of the error bar are asymmetrical due to the non-linearity of the logarithmic scale. As shown in Figure 9, the lower bound may extend well below the nominal value, and as limit case it could be negative infinity if the confidence region includes the zero point (center of the Smith Chart); in this case, the network analyzer will clip the lower bound to about -200 dB.

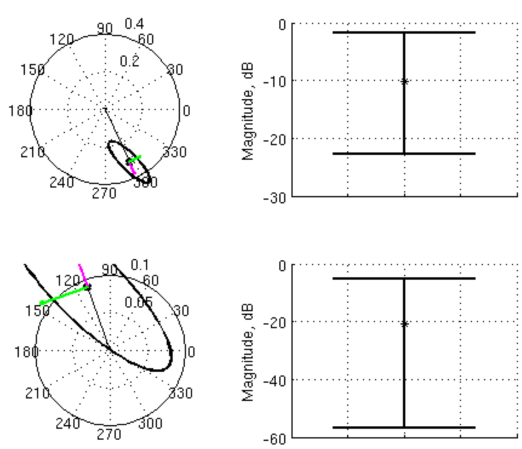


Figure 9: Magnitude uncertainties in dB, showing the asymmetry of the respective error bars.

VNA uncertainty model

The VNA processing model is shown in Figure 10.

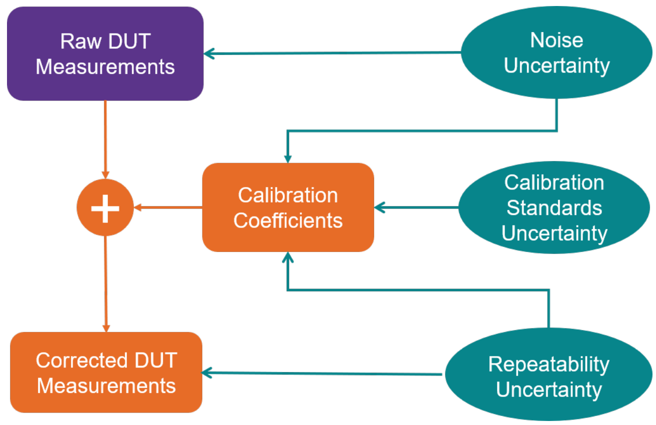
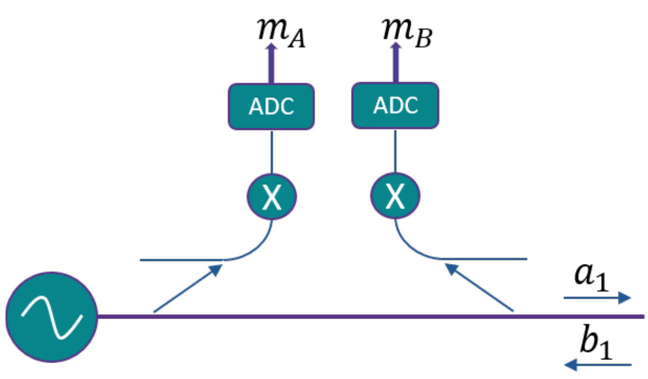


Figure 10: VNA processing and uncertainty model

The raw measurement of your DUT are corrected for the systematic error with a set of error coefficients obtained from the calibration. Within the limit of VNA receiver linearity, the acquired raw data are linearly linked with the waves at each reference plane as shown in Figure 11 for the one port case.



$$\begin{cases} a_1 = c_1 m_A + c_2 m_B \\ b_1 = c_3 m_A + c_4 m_B \end{cases} \quad (9)$$

Figure 11: VNA System Model

The coefficients c_i which are stored in the *calset*, linearly related the raw data measured with the waves at the reference plane as shown in equation 9. If only S-Parameter is required, equation 9 becomes:

$$\Gamma = \frac{b_1}{a_1} = \frac{c_3 m_A + c_4 m_B}{c_1 m_A + c_2 m_B} =$$

$$= \frac{\frac{c_3}{c_4} + \frac{m_B}{m_A}}{\frac{c_1}{c_4} + \frac{c_2}{c_4}\frac{m_B}{m_A}} = \frac{\Gamma_m - E_D}{E_S \Gamma_m + E_R - E_S E_D} \quad (10)$$

Where the relationships among the traditional Directivity E_D,Source Match E_S,Reflection Tracking E_R and measured gamma Γ_M and the simple linear coefficients c_i is given.

The entire process of calibration and measurement is affected by random uncertainty due to several components which interact as shown in Figure10. There are three main sources of uncertainty in this process:

- Measurement noise, due to electrical noise in the VNA receivers and phase noise of the local oscillators
- Repeatability uncertainty, a combined effect due to test cables and connector mating imperfections
- Calibration standards uncertainty, due to imperfect definitions of calibration standards.

Each of these contribution is independently characterized either by Keysight in the case of the calibration standards or through a dedicated procedure managed by the new PNA Uncertainty Manager [3] for the noise and repeatability. Let's review these uncertainty contributions.

Measurement noise

There are two different kind of measurement noise in modern VNAs:

Noise Floor caused by the VNA internal receivers' noise floor, limits the maximum sensitivity. This low-level noise affects the measurement of very lossy devices, like filters in their stopband.

Trace noise also known as jitter noise, due to imperfect coherence of the receivers and the internal RF source; this is caused by the phase noise of the RF and LO (Local Oscillator) signals, and becomes significant when the received signal is high, like in measurements of low-loss devices or filter passband.

Due to their different nature, these two noise contributions are modeled differently. An example of their effect on a simulated filter measurement is shown in Figure 12.

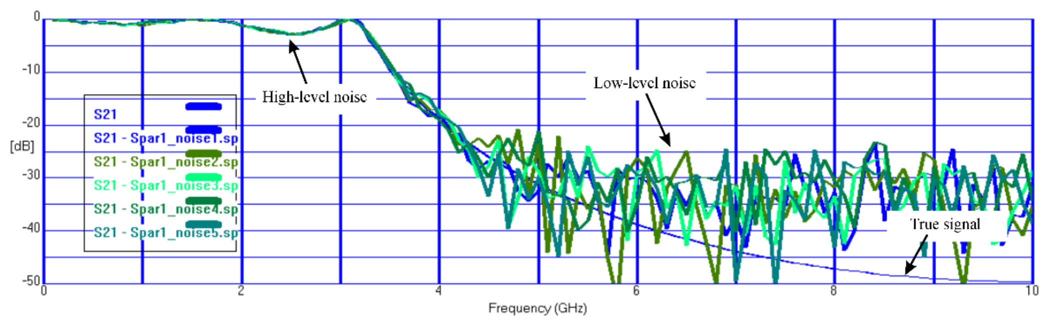


Figure 12: Simulation of the effect of noise floor and trace noise superimposed to ideal data on a Low pass filter measurement

If we consider the noise floor, the measured parameters can be written as

$$S_{m\text{Measurement}} = S_{m0\text{True value}} + \delta n_{L\text{Noise floor}} \tag{11}$$

being a random signal, δn_L cannot be directly measured, but its statistical properties can be estimated from repeated measurements. The trace noise is modeled differently:

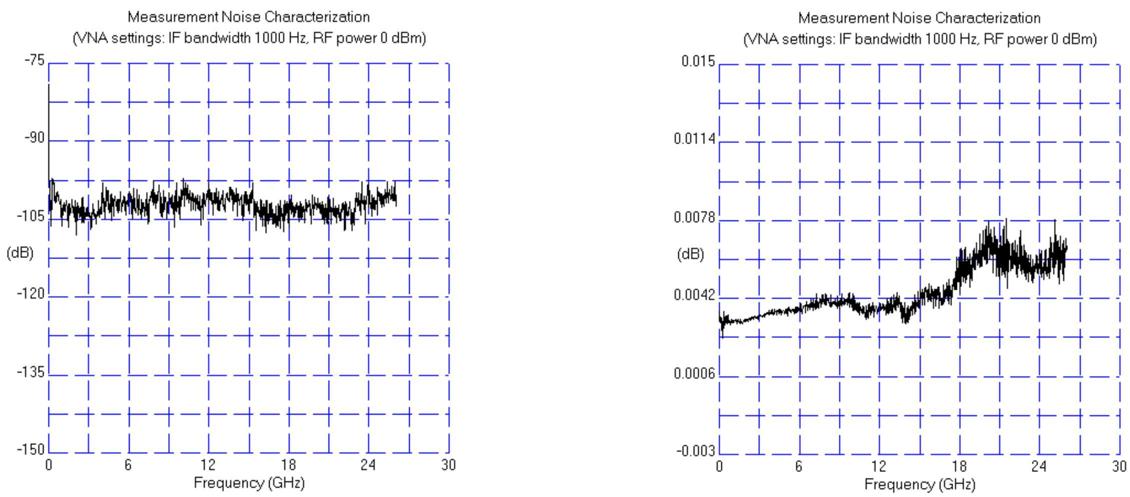
$$S_{m\text{Measurement}} = S_{m0\text{True value}} \left(1 + \delta n_{H\text{Trace noise}}\right) \tag{12}$$

and it can be shown that the trace noise effect is proportional to the measured parameter magnitude and is estimated as well from repeated measurements.

Finally, the two noise sources are combined as

$$S_m = S_{m0} \left(1 + \delta n_H\right) + \delta n_L \tag{13}$$

The statistical properties, i.e. the variances v_L and v_H, of the measurement noise can be computed from a series of repeated measurements, and the Uncertainty Manager, integrated inside the firmware, guides the user to perform in situ noise characterizations of the user PNA. An example of noise characterization is shown in Figure 13.



(a) Noise floor

(b) Trace noise

Figure 13: Typical noise characterization results for a N5222A PNA.

Connector and cable repeatability

High-frequency connections are often sources of error contributions that impact the overall uncertainty. In order to achieve reliable measurements, quality cables must be used, and periodical cleaning and gaging of connectors is important as well. Besides that, there is always an unavoidable repeatability error which is due to small random connectors misalignments, bending test cables or contact resistor effect. Keysight Technologies has developed simple procedures to characterize overall cables and connectors repeatability. The repeatability effect due to both cable and connector interface is modeled as a 2-Port Scattering matrix, connected in between the Network Analyzer and the Device under Test (Figure 14).

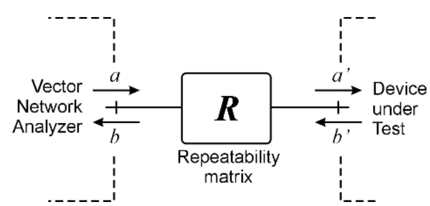


Figure 14: Connector and cable repeatability model.

Assuming a perfect connection, this scattering matrix would be unitary, but, since small imperfections do exist, they are modeled by two random variables which describe the non-deterministic contribution in the transmission (δ_T) and reflection (δ_R) parameters of the repeatability matrix:

$$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} \delta_R & \delta_T \\ \delta_T & \delta_R \end{bmatrix} \tag{14}$$

The new Uncertainty Manager tool has the capability to characterize the repeatability of your own connectors and cables. The characterization procedure requires multiple connections of a Load and a Short standards; each time the standard has to be disconnected and connected again using a proper torque wrench, as one would do for ordinary measurements. The procedure computes the variances and covariances of the repeatability for each PNA port. In the very common case where a test cable is used, the cable should be moved each time the Load or Short standard is connected. In this way one realistically replicates the cable position during the measurements. The number of connections can be selected by the user, but a minimum of 20 connections should be performed to achieve satisfactory results, while 50 connections typically provide near perfect results (Figure 15).

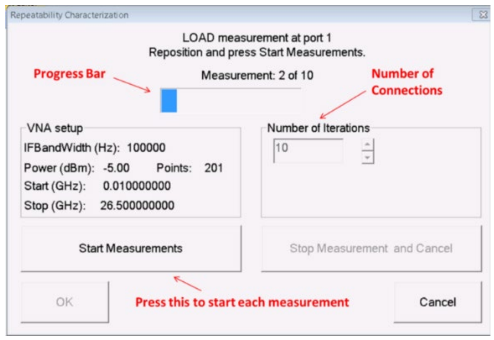
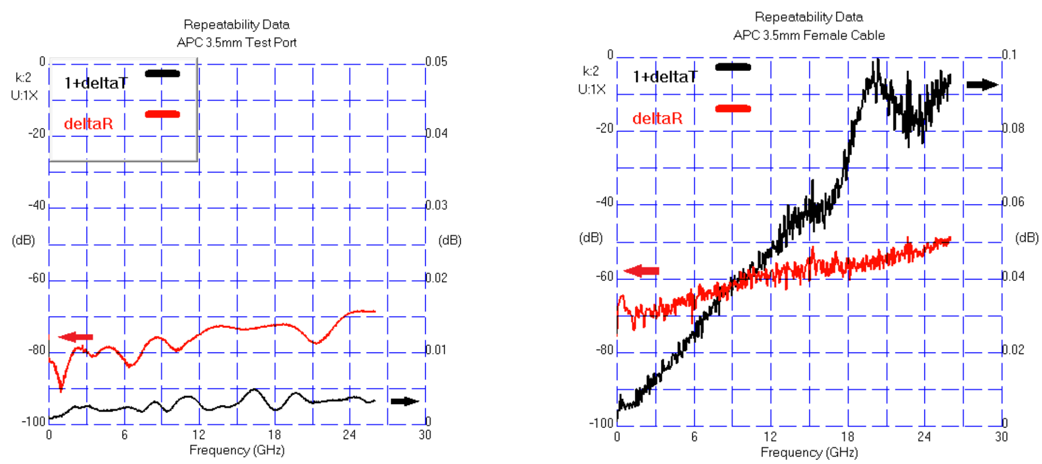


Figure 15: The connector repeatability characterization dialog in Uncertainty Manager.

The results of a typical characterization of a 3.5 mm connector are shown in Figure 16, where the plot shows the transmission repeatability variance of both δ_R and δ_T parameters. If the same test is repeated with a flexible cable as shown in Figure16b, the results show a much higher variance compared to the connector only case (Figure 16a), which practically means that reflection coefficients below -50 dB cannot be measured reliably with this cable and connector setup above 12GHz.



a) The 3.5 mm connector repeatability parameters

b) The 3.5 mm connector cable repeatability parameters

Figure 16: cable repeatability example

Calibration Uncertainty

The calibration uncertainty not only depends on the noise and the repeatability, but also on the uncertainty of the standards used during the calibration process. For this reason, each Keysight Calkit has been update and uncertainty information has been added as database standard as shown in Figure 17.

The same linear correction model of equation 9 is used when power measurement is required. Since the traditional S-Parameter calibration provides only three of the four error terms, an additional *power calibration* is needed to obtain all the c_i.

This power calibration normally consists on the insertion of a power meter in place of the DUT. By a direct comparison of the power meter and VNA reading the forth calibration coefficient is readily obtained.

Clearly the uncertainty of the additional power meter measurement affects any further power measurement taken by the VNA. For this reason, the characterization of Keysight power meter uncertainty is now included in the firmware. The user simply selects the uncertainty characteristic of the sensor used during the power calibration with the new power sensor uncertainty dialog shown in Figure 18.

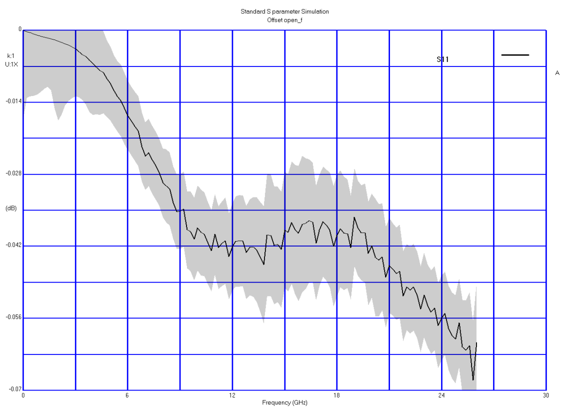


Figure 17: Database Standard (S11) example with uncertainty area

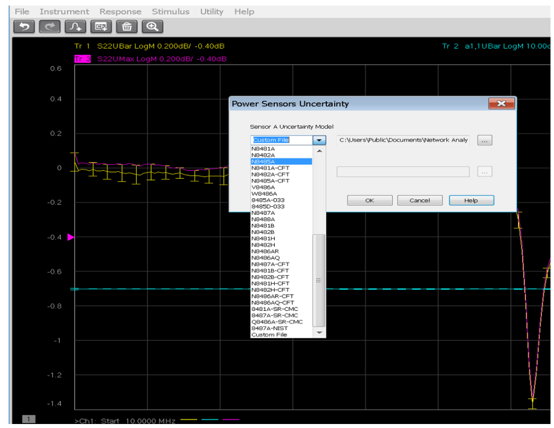


Figure 18: Power Sensor uncertainty dialog

Uncertainty evaluation process

To properly obtain uncertainty information a consistent process should be followed. The main steps are:

1. **Perform the Noise Characterization with the same cable and IF bandwidth you will use for the actual measurement.** Although the firmware will automatically compensate for a different IF bandwidth, different IF band imply slightly different frequency response thus different noise contribution.

2. **Use the exact same cable and connector you characterized.** Each cable and connector pair have a slightly different repeatability characteristic, to obtain the best results always used the cable you have characterized.

3. **Perform the DUT measurement right after the calibration.** The intent of the new uncertainty capability is to provide the best evaluation achievable of the random effects involved in the measurement process. If time passes from the calibration to the actual measurement, the drift of the equipment may impact the final result. For this reason, the most accurate measurement is obtained right after the S-Parameter and power calibration is performed.

4. **Use proper cable and connector care.** A low-quality cable, lack of connector cleaning or improper use of torque wrenches may dramatically impair the overall results. These *old school* suggestions still offer the best path for a low uncertainty measurement at microwave frequencies.

Once the VNA is calibrated a typical result of an uncertainty measurement is shown in Figure 19. The new firmware has the new trace uncertainty property dialog of Figure 20, which lets the user change the graphic appearance of the uncertainty (error bars or shade area) as well as the different contributions. By activating the different uncertainty components, the user can easily understand where potential problems come from. If for example the noise contribution is predominant a reduced IF bandwidth may be required.

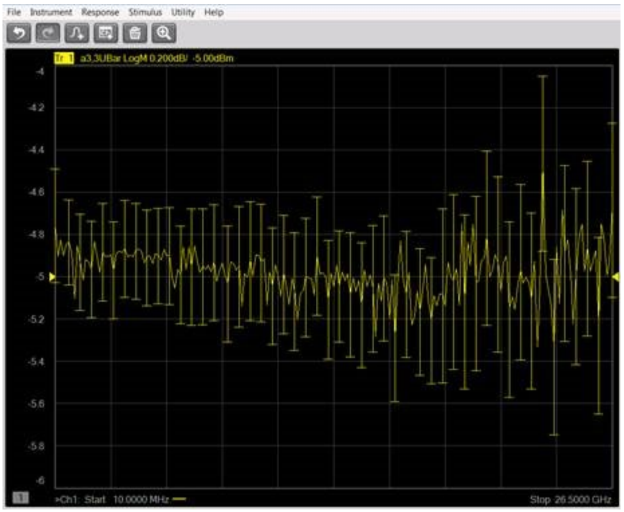


Figure 19: Example of power measurement with uncertainty

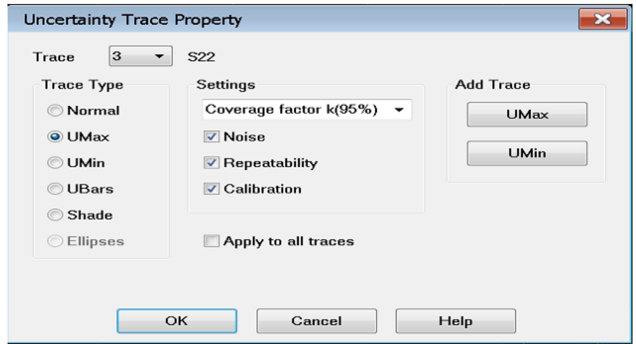


Figure 20: Trace uncertainty dialog

An example of uncertainty data usage

Beside the direct uncertainty information obtainable from the PNA with the new S93015B software option, the availability of wave and S-Parameter uncertainty data opens up a whole new capability to the modern testing engineer.

For example, suppose we wish to compute the Operating Power Gain of an amplifier from its S-Parameters. The well-known formulas for computing the gain is:

$$G_p = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{in}|^2)} \qquad (15)$$

But if the measurement is affected by uncertainty what is the influence on our derived gain?

Prior to the introduction of S93015B software option, to answer this question a cumbersome yet non-correlated computation was required which often results in a simple RSS (Root Squared Sum) overestimate uncertainty.

With the new uncertainty data coupled with MATLAB™ and the METAS™ uncertainty library [4], computing the uncertainty on derived quantities become straightforward.

Here it's the procedure to follow:

1. Calibrate the PNA/PNA-X with the uncertainty flag on.
2. Perform the measurement of the amplifier.
3. Save the S-Parameter data with the new **.unc* file format.
4. Load the data in MATLAB with the routine available on the Keysight support website [6].
5. Compute and plot the GAIN with its uncertainty.

The overall MATLAB script is available on Keysight support and the final result is shown in Figure 21.

A once complicate process is reduced to writing few lines of code thank to the new S93015B software option and the supporting routines.

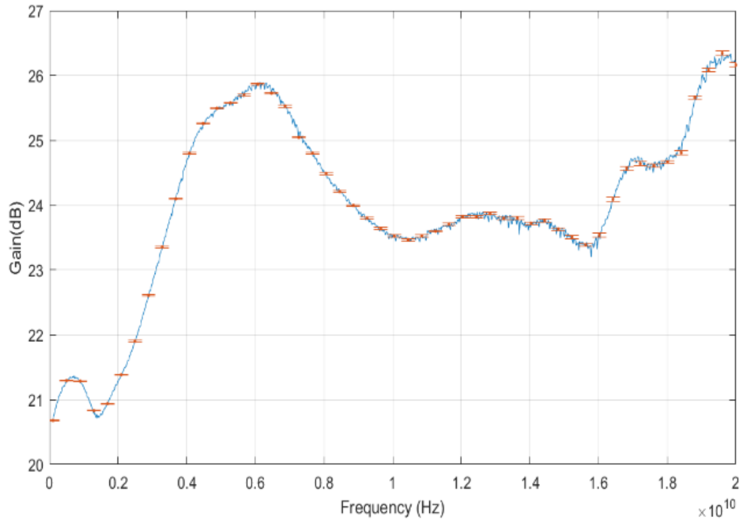


Figure 21: Operating Power Gain with fully correlated uncertainty

Conclusion

The new PNA/ PNA-X real time uncertainty option (S93015B) offers a complete capability to better investigate the DUT measurement. The possibility to plot and use the uncertainty data gives the user a new tool to understand and identify the DUT problems and the measurement issues in a clear and simple way. The new data allows the user to easily compute the uncertainty on derived measurements which will be fundamental to all the applications where PNA measurements are used.

APPENDIX

The new file formats associated with uncertainty

The Real Time Uncertainty option (S93015B) introduces four different new file formats when measurement uncertainty is saved:.

1. ***.sdatcv** This format is the METAS format for S-Parameter measurement data. This format described in [5] stores the S-Parameter and their variance matrix.
2. ***.u*p** This format contains the S-Parameter uncertainty without correlation matrix. It is useful when measurement reports are required.
3. ***.unc** Unified parametric file format. This format stores the expression on nominal values and uncertainty contributions for any set of output parameters which the PNA can compute the uncertainty of. It allows to compute correlation among different measurements when saved with this file format.
4. ***.dsd** Uncertainty calibration standard. This is the format used to specify a measurement which can be used as calibration standard with uncertainty.

References

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